Summary of key points

1 The integral $\int_a^b f(x)\,dx$ is **improper** if either:

- one or both of the limits is infinite
- $f(x)$ is undefined at $x = a$, $x = b$ or another point in the interval $[a, b]$.

2 The **mean value** of the function $f(x)$ over the interval $[a, b]$, is given by

$$\boldsymbol{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

3 If the function $f(x)$ has mean value $\bar{f}$ over the interval $[a, b]$, and k is a real constant, then:

- $f(x) + k$ has mean value $\bar{f} + k$ over the interval $[a, b]$
- $kf(x)$ has mean value $k\bar{f}$ over the interval $[a, b]$
- $-f(x)$ has mean value $-\bar{f}$ over the interval $[a, b]$.

4
- $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

5
- $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c,\ a > 0,\ |x| < a$
- $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + c$