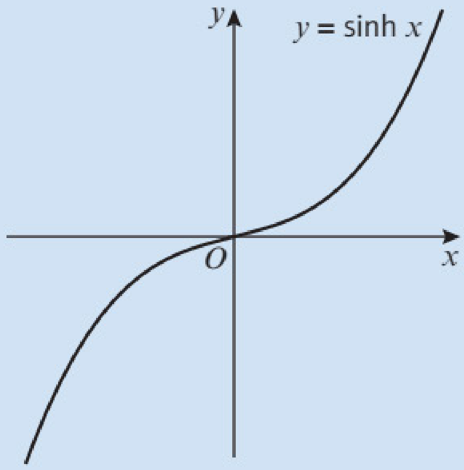
Summary of key points

1 • Hyperbolic sine (or **sinh**) is defined as $\sinh x \equiv \frac{e^x - e^{-x}}{2}, x \in \mathbb{R}$

• Hyperbolic cosine (or **cosh**) is defined as $\cosh x \equiv \frac{e^x + e^{-x}}{2}, x \in \mathbb{R}$

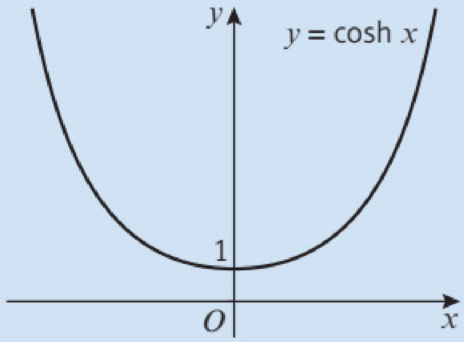
• Hyperbolic tangent (or **tanh**) is defined as $\tanh x \equiv \frac{\sinh x}{\cosh x} \equiv \frac{e^{2x} - 1}{e^{2x} + 1}, x \in \mathbb{R}$

2 • The graph of $y = \sinh x$:



For any value a, $\sinh(-a) = -\sinh a$.

• The graph of $y = \cosh x$:



For any value a, $\cosh(-a) = \cosh a$.

3 The table shows the inverse hyperbolic functions, with domains restricted where necessary.

Hyperbolic function	Inverse hyperbolic function
$y = \sinh x$	$y = \text{arsinh}\, x$
$y = \cosh x, x \geqslant 0$	$y = \text{arcosh}\, x, x \geqslant 1$
$y = \tanh x$	$y = \text{artanh}\, x, \lvert x \rvert < 1$

4 • $\text{arsinh}\, x = \ln\left(x + \sqrt{x^2 + 1}\right)$ • $\text{arcosh}\, x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geqslant 1$ • $\text{artanh}\, x = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right), \lvert x \rvert < 1$

5 $\cosh^2 A - \sinh^2 A \equiv 1$

6 • $\sinh(A \pm B) \equiv \sinh A \cosh B \pm \cosh A \sinh B$

• $\cosh(A \pm B) \equiv \cosh A \cosh B \pm \sinh A \sinh B$

7 • $\frac{d}{dx}(\sinh x) = \cosh x$ • $\frac{d}{dx}(\cosh x) = \sinh x$ • $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

• $\frac{d}{dx}(\text{arsinh}\, x) = \frac{1}{\sqrt{x^2 + 1}}$ • $\frac{d}{dx}(\text{arcosh}\, x) = \frac{1}{\sqrt{x^2 - 1}}$ • $\frac{d}{dx}(\text{artanh}\, x) = \frac{1}{1 - x^2}$

8 • $\int \sinh x \, dx = \cosh x + c$ • $\int \cosh x \, dx = \sinh x + c$

9 $\int \tanh x \, dx = \ln \cosh x + c$

10 • $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \text{arsinh}\left(\frac{x}{a}\right) + c$ • $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \text{arcosh}\left(\frac{x}{a}\right) + c, \ x > a$