Summary of key points

1 You can solve a first-order differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ by multiplying every term by the **integrating factor** $e^{\int P(x)dx}$.

2 The natures of the roots α and β of the **auxiliary equation** determine the **general solution** to the second-order differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + c = 0$.

You need to consider three different cases:

- **Case 1: $b^2 > 4ac$**

  The auxiliary equation has two real roots α and β ($\alpha \neq \beta$). The general solution will be of the form $y = Ae^{\alpha x} + Be^{\beta x}$ where A and B are arbitrary constants.

- **Case 2: $b^2 = 4ac$**

  The auxiliary equation has one repeated root α. The general solution will be of the form $y = (A + Bx)e^{\alpha x}$ where A and B are arbitrary constants.

- **Case 3: $b^2 < 4ac$**

  The auxiliary equation has two complex conjugate roots α and β equal to $p \pm q$i. The general solution will be of the form $y = e^{px}(A\cos qx + B\sin qx)$ where A and B are arbitrary constants.

3 A **particular integral** is a function which satisfies the original differential equation.

4 To find the general solution to the differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$,

- Solve the corresponding homogeneous equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ to find the complementary function, C.F.
- Choose an appropriate form for the particular integral, P.I., and substitute into the original equation to find the values of any coefficients.
- The general solution is $y =$ C.F. + P.I.