Summary of key points

1 If a particle starts from the point with position vector $\mathbf{r}_0$ and moves with constant velocity $\mathbf{v}$, then its displacement from its initial position at time t is $\mathbf{v}t$ and its position vector $\mathbf{r}$ is given by $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$.

2 For an object moving in a plane with constant acceleration:

- $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
- $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

where

- $\mathbf{u}$ is the initial velocity
- $\mathbf{a}$ is the acceleration
- $\mathbf{v}$ is the velocity at time t
- $\mathbf{r}$ is the displacement at time t.

Vectors in kinematics

Velocity and **displacement** are both **vector quantities**. This means they have **magnitude** and **direction**. You can use **unit vectors** $\mathbf{i}$ and $\mathbf{j}$ to describe the position or velocity of an object.

Position and velocity

Make sure you don't confuse position vectors and velocity vectors:

1 **Position** (or **displacement**) **vectors** tell you where an object is relative to a **fixed origin**.

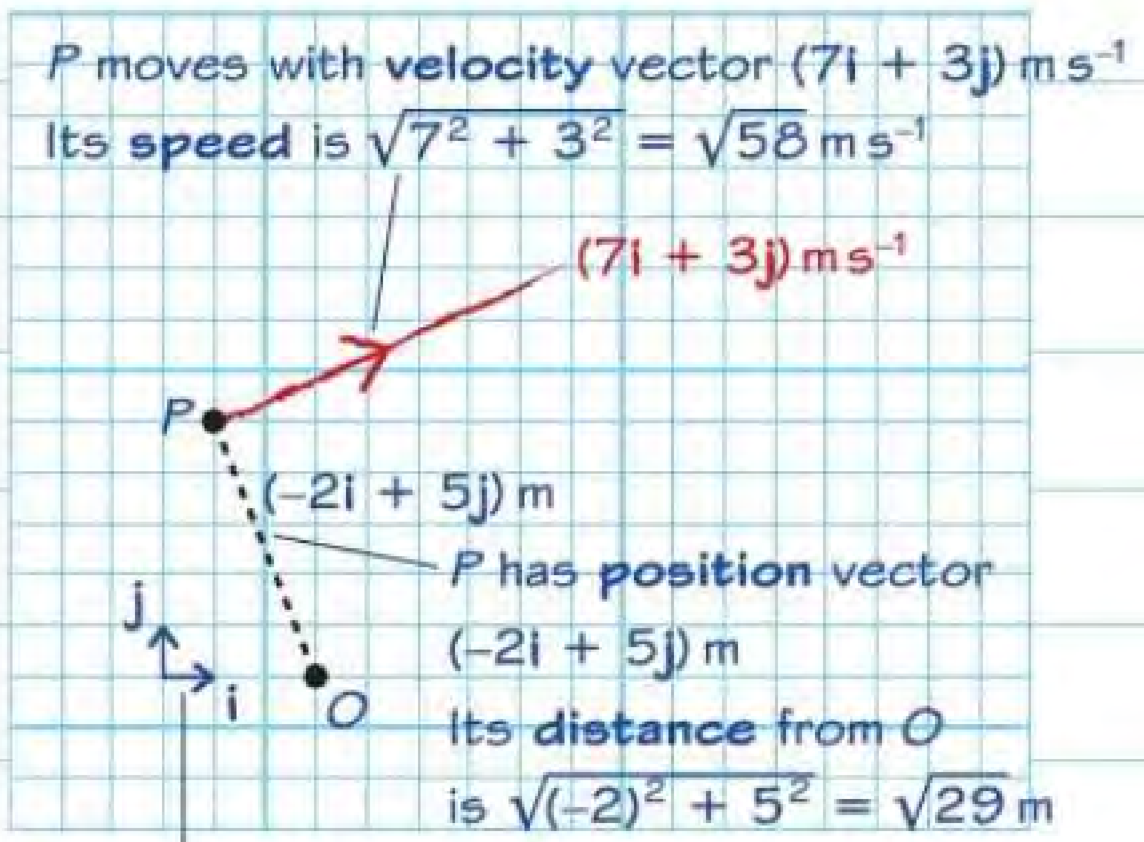
Distance is the **magnitude** of a position vector.

If an object has position vector $\mathbf{r} = (a\mathbf{i} + b\mathbf{j})$ m, its distance from the origin is $|\mathbf{r}| = \sqrt{a^2 + b^2}$ m

2 **Velocity vectors** tell you what direction an object is moving in and how fast.

Speed is the **magnitude** of a velocity vector.

If an object has velocity vector $\mathbf{v} = (p\mathbf{i} + q\mathbf{j})\,\mathrm{m\,s^{-1}}$, its speed is $|\mathbf{v}| = \sqrt{p^2 + q^2}\,\mathrm{m\,s^{-1}}$



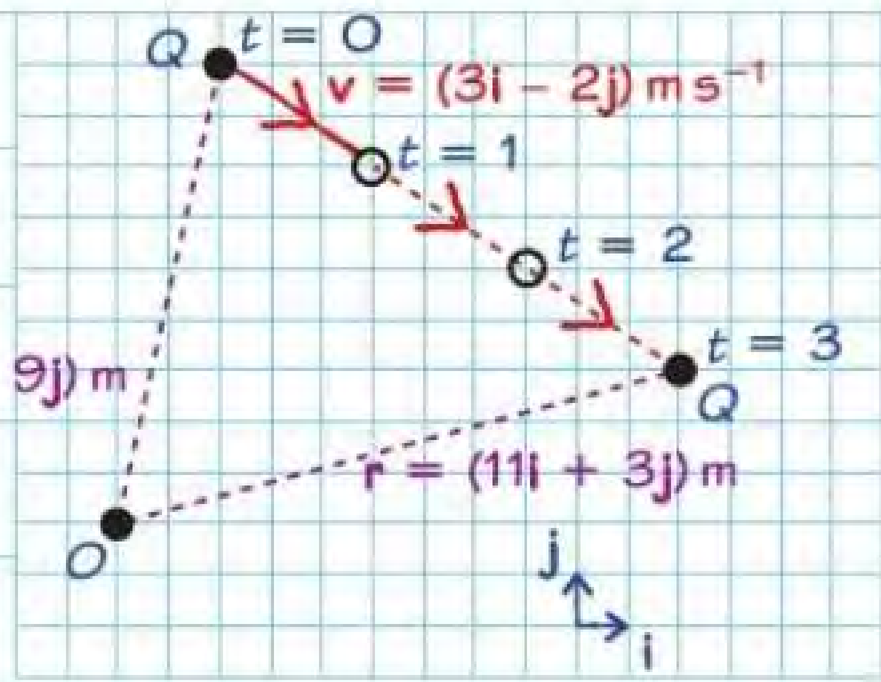
Velocity and time

If an object starts at a point with position vector $\mathbf{r}_0$ m and moves for time t seconds with velocity vector $\mathbf{v}$ then its new position vector $\mathbf{r}$ will be given by

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$$

In the diagram, Q starts with initial position vector $\mathbf{r}_0 = (2\mathbf{i} + 9\mathbf{j})$ m and moves with velocity vector $\mathbf{v} = (3\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. After 3 seconds:

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
$= (2\mathbf{i} + 9\mathbf{j}) + 3(3\mathbf{i} - 2\mathbf{j})$
$= (2 + 3 \times 3)\mathbf{i} + (9 - 3 \times 2)\mathbf{j}$
$= 11\mathbf{i} + 3\mathbf{j}$



Worked example

A particle P is moving with constant velocity $(-5\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$. At time $t = 9$ seconds, P is at the point with position vector $(-12\mathbf{i} - 2\mathbf{j})$ metres. Find the distance of P from the origin at time 5 s. **(5 marks)**

$\underset{\sim}{r} = \underset{\sim}{r_0} + \underset{\sim}{v}t$

$(-12\underset{\sim}{i} - 2\underset{\sim}{j}) = \underset{\sim}{r_0} + 4(-5\underset{\sim}{i} + 4\underset{\sim}{j})$

$\underset{\sim}{r_0} = (-12\underset{\sim}{i} - 2\underset{\sim}{j}) - 4(-5\underset{\sim}{i} + 4\underset{\sim}{j})$

$= 8\underset{\sim}{i} - 18\underset{\sim}{j}$

$|\underset{\sim}{r_0}| = \sqrt{8^2 + (-18)^2} = 19.7$ m (3 s.f.)

You know the position of P at $t = 9$ s and you want to know its position 4 seconds earlier. Let $\mathbf{r}_0$ be the position of P at 5 s, and use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ with $t = 4$, $\mathbf{v} = (-5\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ and $\mathbf{r} = (-12\mathbf{i} - 2\mathbf{j})$ m.

The **distance** from the origin is the **magnitude** of the vector. $|\mathbf{r}_0| = \sqrt{a^2 + b^2}$

Now try this

A ship S moves with constant velocity $(-10\mathbf{i} - 5\mathbf{j})\,\mathrm{km\,h^{-1}}$. It passes a buoy with position vector $(12\mathbf{i} + 25\mathbf{j})$ km at midday. Find

(a) the speed of the ship Answer **(2 marks)**

(b) its position vector at 3 pm. Answer **(3 marks)**

Velocity and position vectors have **units**. The units of velocity here are $\mathrm{km\,h^{-1}}$ so measure time in hours.

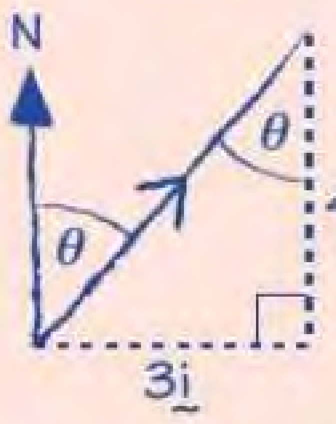
Vectors and bearings

If you have to solve a vector problem involving bearings, **i** will be the unit vector **due east** and **j** will be the unit vector **due north**.

Worked example

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{km h}^{-1}$ and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})\,\text{km h}^{-1}$.

(a) Find, to the nearest degree, the bearing on which Q is moving. **(2 marks)**



$\tan\theta = \frac{3}{4}$ so $\theta = 36.86...°$

Bearing is 037° (nearest degree)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km. At time t hours after 2 pm, the position vector of P is **p** km and the position vector of Q is **q** km.

(b) Write down expressions, in terms of t, for

(i) **p**

(ii) **q**

(iii) $\overrightarrow{PQ}$. **(5 marks)**

(i) $\underset{\sim}{p} = (\underset{\sim}{i} + \underset{\sim}{j}) + t(2\underset{\sim}{i} - 3\underset{\sim}{j})$

(ii) $\underset{\sim}{q} = (-2\underset{\sim}{j}) + t(3\underset{\sim}{i} + 4\underset{\sim}{j})$

(iii) $\underset{\sim}{q} - \underset{\sim}{p} = [(-2\underset{\sim}{j}) + t(3\underset{\sim}{i} + 4\underset{\sim}{j})]$
$\quad - [(\underset{\sim}{i} + \underset{\sim}{j}) + t(2\underset{\sim}{i} - 3\underset{\sim}{j})]$
$\quad = (-\underset{\sim}{i} - 3\underset{\sim}{j}) + t(\underset{\sim}{i} + 7\underset{\sim}{j})$

(c) Find the time when

(i) Q is due north of P

(ii) Q is north-west of P. **(4 marks)**

$\overrightarrow{PQ} = (-\underset{\sim}{i} - 3\underset{\sim}{j}) + t(\underset{\sim}{i} + 7\underset{\sim}{j})$
$= (-1 + t)\underset{\sim}{i} + (-3 + 7t)\underset{\sim}{j}$

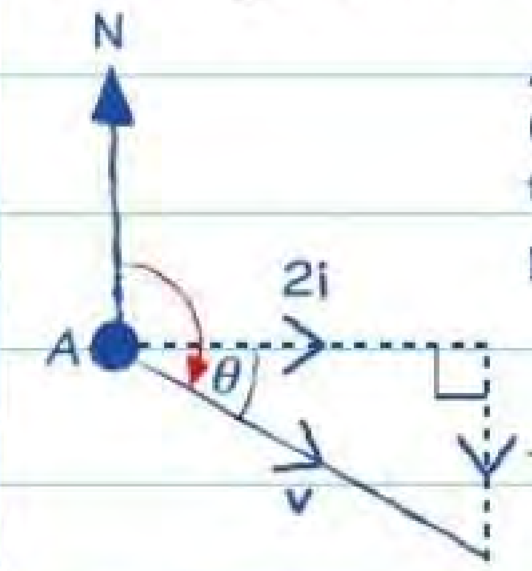
(i) $\underset{\sim}{i}$ component = 0
$-1 + t = 0$
$t = 1$, so time is 3 pm

(ii) $\underset{\sim}{i}$ component = $-\underset{\sim}{j}$ component
$-1 + t = -(-3 + 7t)$
$8t = 4$
$t = 0.5$, so time is 2.30 pm

In part (b), find expressions for the position vectors of A and B, $\mathbf{r}_A$ and $\mathbf{r}_B$ in terms of t. To show that A and B will **collide** you need to find a single value of t that makes the **i** components equal **and** the **j** components equal.

Finding a bearing

You can use trigonometry to find the bearing an object is moving on if you know its velocity vector. Remember that bearings are measured **clockwise from north** and you should always give bearings to the **nearest degree**.



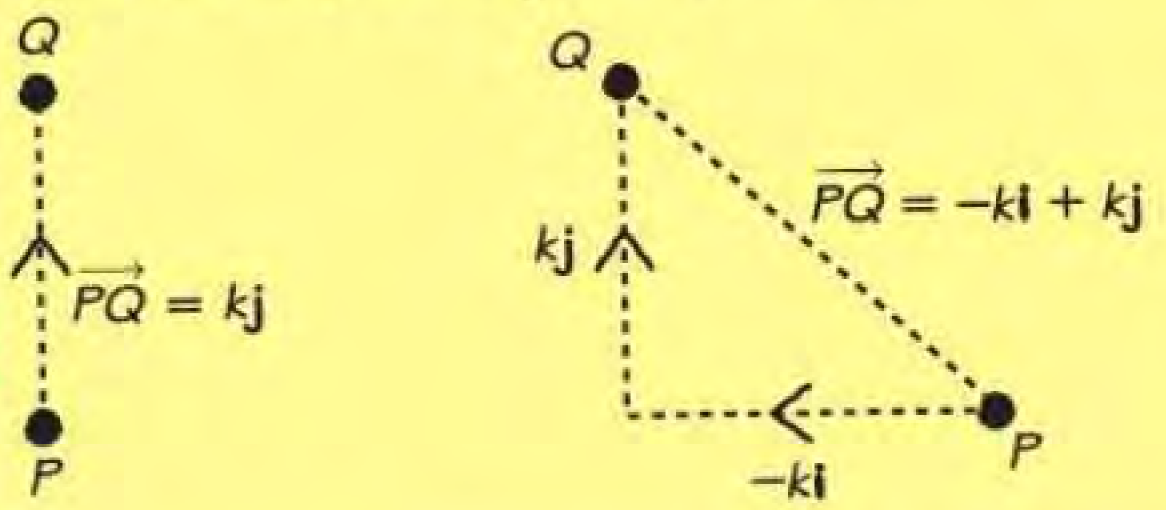
A is moving with velocity vector $(2\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

$\tan\theta = \frac{1}{2}$ so $\theta = 26.56...°$

Bearing $= 90° + 26.56...°$
$= 117°$ (nearest degree)

For part (b), use $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ to write expressions for the position vectors of P and Q in terms of t.

For part (c), write the vector $\overrightarrow{PQ}$ in terms of its **i** component and its **j** component. When Q is due north of P the **i** component of $\overrightarrow{PQ}$ will be 0. When Q is north-west of P the **i** component of $\overrightarrow{PQ}$ will be **minus** the **j** component.



Now try this

A particle A moves with constant velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$. At time $t = 0$, A is at the point with position vector $(3\mathbf{i} - 8\mathbf{j})$ m.

(a) Find the direction in which A is moving, giving your answer as a bearing.

Answer **(2 marks)**

At time $t = 0$, a second particle B is at the point with position vector $(-21\mathbf{i} + 16\mathbf{j})$ m, and is moving with velocity $(5\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$.

(b) Show that A and B will collide at a point P and find the position vector of P.

Answer **(5 marks)**