

Resolving forces

You can answer some questions by resolving forces into **perpendicular components**.

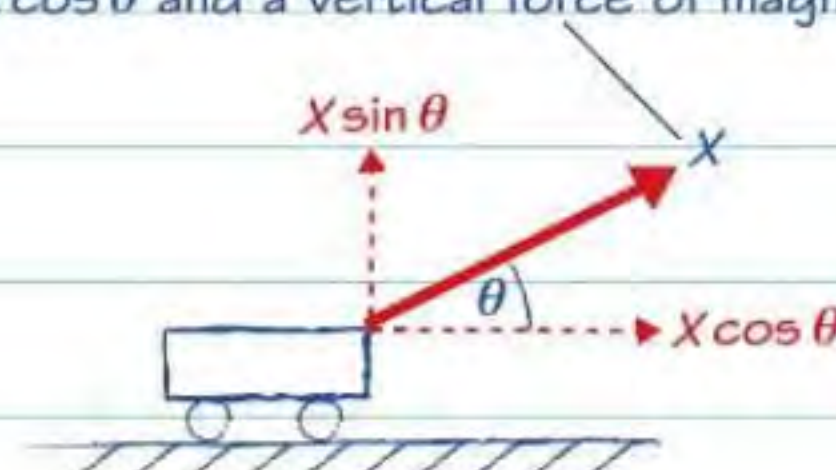
Using sin and cos

To resolve a force into components, imagine sketching a right-angled triangle, with the force as its hypotenuse. For example, a force of magnitude X acting at an angle θ to the horizontal can be resolved **horizontally** and **vertically**:

$$R(\rightarrow): X \cos \theta \quad R(\uparrow): X \sin \theta$$

You can use the notation $R(\rightarrow)$ and $R(\uparrow)$ to show which direction you are resolving in. The arrow points in the **positive** direction.

This force has the same effect as a horizontal force of magnitude $X \cos \theta$ and a vertical force of magnitude $X \sin \theta$.

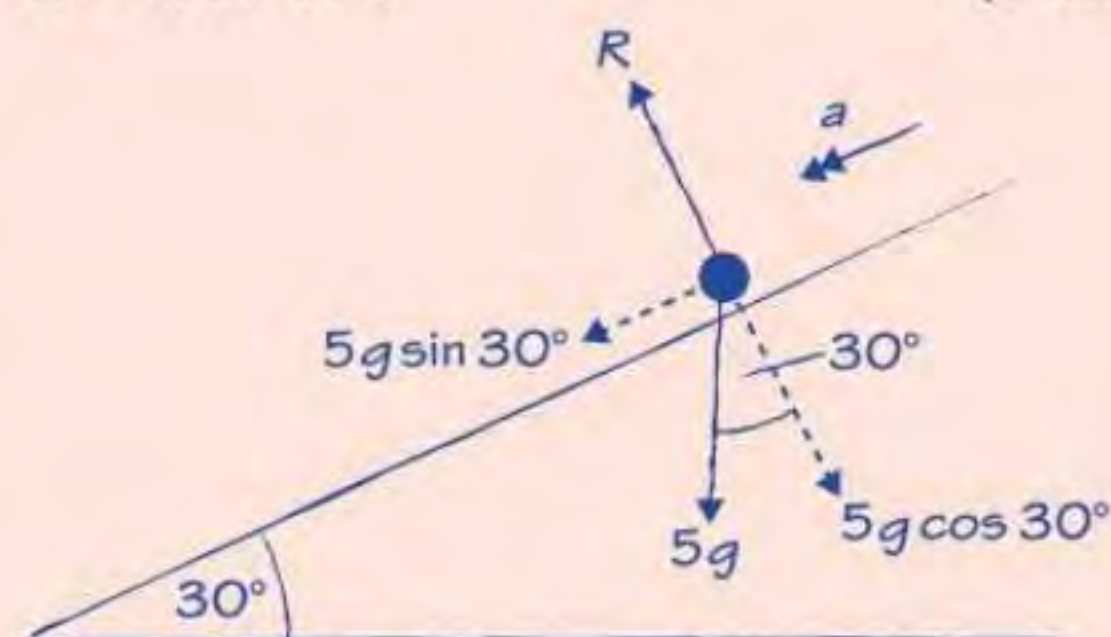


$X \cos \theta$ is **adjacent** to the angle, and $X \sin \theta$ is **opposite** the angle.

Worked example

A box of mass 5 kg is lying on a smooth plane which is inclined at an angle of 30° to the horizontal. The box is released from rest and accelerates at $a \text{ m s}^{-2}$ down the slope. Modelling the box as a particle, find

(a) the value of a (3 marks)



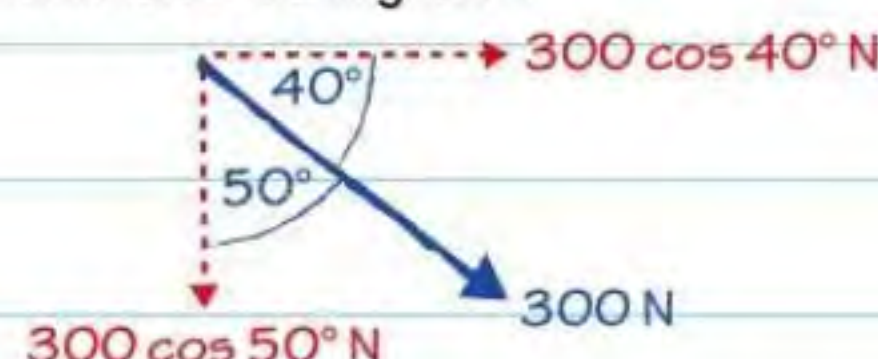
$$\begin{aligned} R(\downarrow): 5g \sin 30^\circ &= 5a \\ 2.5g &= 5a \\ a &= 0.5g = 4.9 \text{ m s}^{-2} \end{aligned}$$

(b) the magnitude of the normal reaction of the slope on the box. (2 marks)

$$\begin{aligned} R(\nearrow): R - 5g \cos 30^\circ &= 0 \\ R &= \frac{5\sqrt{3}}{2}g = 42 \text{ N (2 s.f.)} \end{aligned}$$

Just cos

If the direction you are resolving in makes an angle of θ with the line of action of the force F , then the component in that direction is $F \cos \theta$. You can use this to resolve without using sin.



Only the component of the force which acts **down the slope** contributes to the acceleration of the box. You need to resolve the **weight** of the box ($5g \text{ N}$) into components **parallel** and **perpendicular** to the slope. Use $R(\downarrow)$ to show that the positive direction is **down** the slope.

The **normal reaction** always acts **perpendicular** to the plane. It is **equal in magnitude** to the component of the weight that is perpendicular to the plane, and acts in the **opposite** direction.

Now try this

A truck is pulling a car using a light, inextensible tow-rope. The tow-rope is at an angle of 40° to the road. Together, the truck and car accelerate constantly at a rate of 2.5 m s^{-2} . The car has mass 800 kg and experiences a constant resistance to motion of magnitude 500 N. Find

(a) the tension in the tow-rope (4 marks)
(b) the normal reaction of the road on the car. (2 marks)

The car is being accelerated by the **horizontal** component of the tension in the tow-rope.

