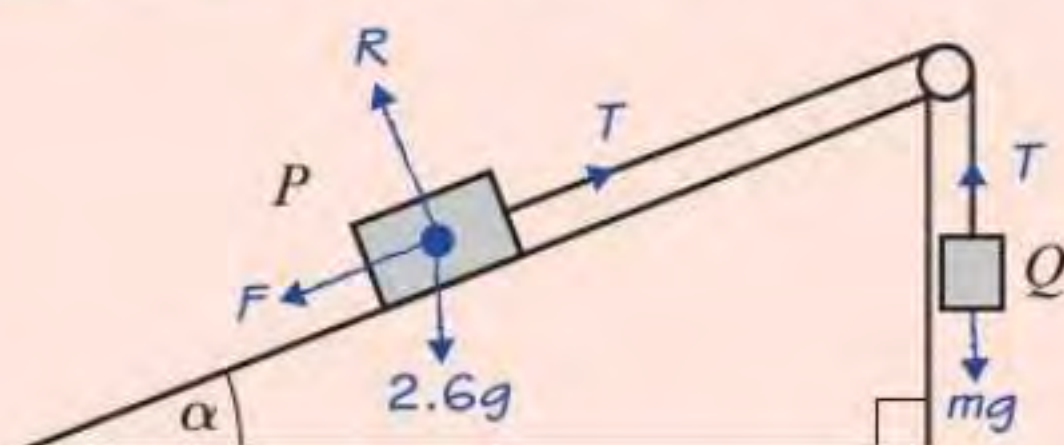


Connected particles 2

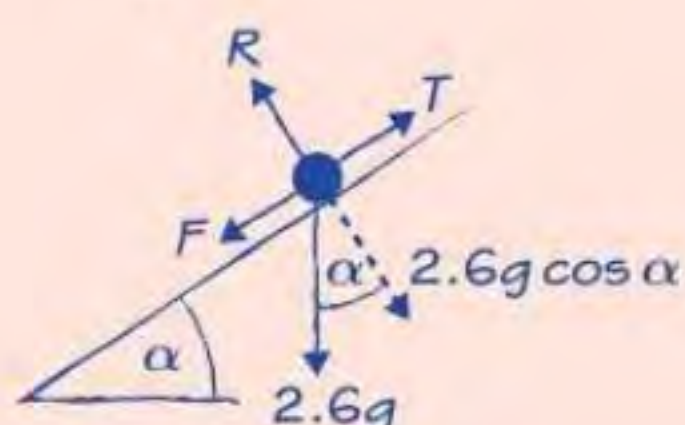
When two particles are connected via a pulley, one particle might be on a **plane**. If the plane is **rough**, you'll have to consider **friction** as well. Have a look at page 168 for a reminder.

Worked example



Two particles P and Q have masses 2.6 kg and $m \text{ kg}$ respectively. The particles are attached to the ends of a light inextensible string that passes over a smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{5}{12}$. The coefficient of friction between P and the plane is 0.5 . The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut. The system is released from rest and Q accelerates vertically downwards at 1.8 m s^{-2} . Find

- (a) the magnitude of the normal reaction of the inclined plane on P (2 marks)



$$R(\perp): R - 2.6g \cos \alpha = 0$$

$$\text{Using } \cos \alpha = \frac{12}{13}: R = 2.6g \times \frac{12}{13} = 24 \text{ N (2 s.f.)}$$

- (b) the value of m . (8 marks)

$$R(\nearrow): \text{Using } F = ma \text{ for } P \text{ to find } T:$$

$$T - F - 2.6g \sin \alpha = 2.6 \times 1.8$$

$$\text{Using } \sin \alpha = \frac{5}{13}: T - F = 14.48 \quad (1)$$

$$F = \mu R = 0.5 \times 24 = 12$$

$$\text{Substituting into (1): } T - 12 = 14.48 \\ T = 26.48 \text{ N}$$

$$R(\downarrow): \text{Using } F = ma \text{ on } Q:$$

$$mg - T = 1.8m$$

$$9.8m - 26.48 = 1.8m$$

$$8m = 26.48$$

$$m = 3.31 = 3.3 \text{ kg (2 s.f.)}$$

You have been given $\tan \alpha = \frac{5}{12}$. You can sketch a right-angled triangle to work out exact values of $\sin \alpha$ and $\cos \alpha$:

$$\sqrt{12^2 + 5^2} = 13 \quad \sin \alpha = \frac{5}{13} \\ \cos \alpha = \frac{12}{13}$$

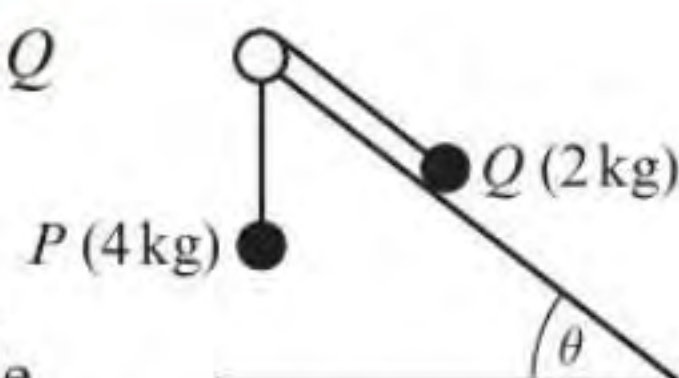
You might also be able to find these fractions on your calculator:

$$\sin\left[\tan^{-1}\left(\frac{5}{12}\right)\right] = \frac{5}{13}$$

Resolve **parallel** to the plane to write an equation of motion for P , and resolve **vertically** to write an equation of motion for Q .

Now try this

Two particles P and Q of masses 4 kg and 2 kg respectively are attached to the ends of a light inextensible string, which runs over a smooth pulley. Q is held at rest on a rough plane inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$. The coefficient of friction between Q and the plane is $\frac{1}{4}$. The particles are released from rest and Q accelerates up the plane.



- (a) Find the acceleration of Q . (10 marks)

After 0.6 s , particle P hits the floor and remains there. Particle Q continues up the slope, reaching its highest point after a further $T \text{ s}$. In this motion Q does not reach the pulley.

- (b) Find the value of T . (6 marks)

You need to use the **suvat** formulae for part (b). u will be the speed of Q after 0.6 s , and v will be 0 . Have a look at pages 150 and 151 for a reminder.