Summary of key points

2 The maximum value of the frictional force $F_{MAX} = \mu R$ is reached when the body you are considering is on the point of moving. The body is then said to be in limiting equilibrium.

3 In general, the force of friction F is such that $F \leqslant \mu R$, and the direction of the frictional force is opposite to the direction in which the body would move if the frictional force were absent.

Friction

Friction is a force which acts in the **opposite** direction to the motion of a particle on a **rough** surface.

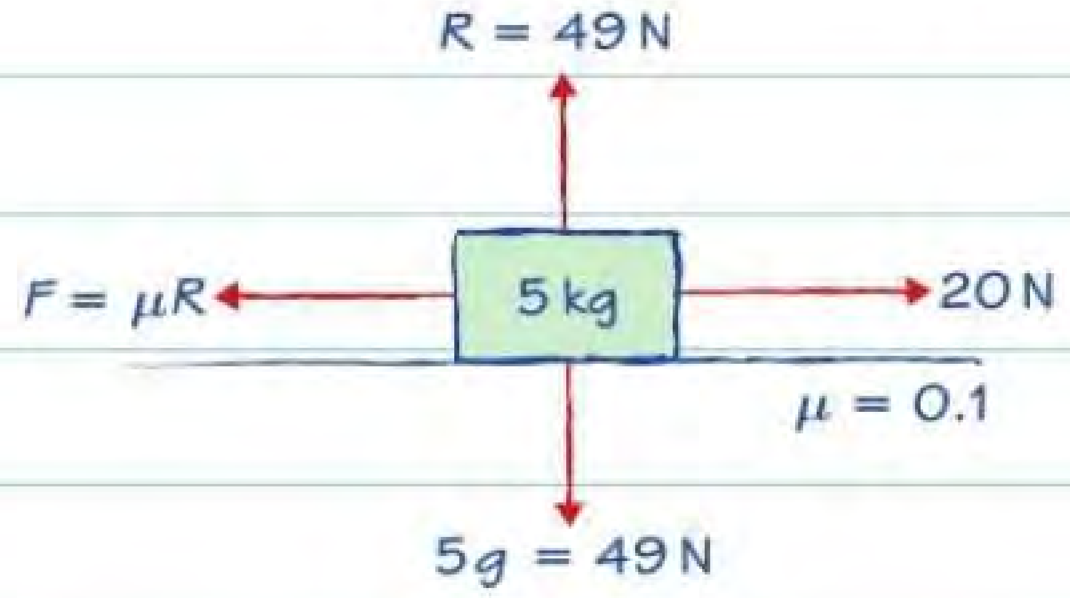
Coefficient of friction

The coefficient of friction, μ, between an object and a surface is a measure of roughness. For a **moving** object the force of friction has magnitude

$$F = \mu R$$

where R is the magnitude of the **normal** reaction between the object and the surface.

If $\mu = 0$ the surface is **smooth** – there is **no** friction. Any surface with $\mu > 0$ is called a **rough** surface.

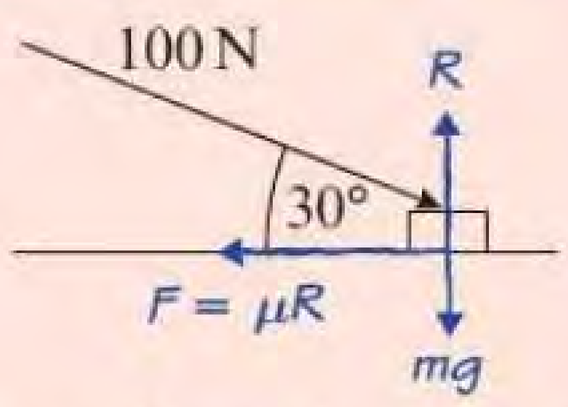


This box is accelerating along a rough surface. The coefficient of friction between the box and the surface is 0.1. The normal reaction is 49 N so

$F = \mu R = 0.1 \times 49 = 4.9$ N

So the resultant force on the box is $20 - 4.9 = 15.1$ N.

Worked example



Everything in blue is part of the answer.

A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor.

Given that the box moves with constant speed, find the mass of the box. **(7 marks)**

R($\rightarrow$): $100\cos 30° - F = 0$

$F = 50\sqrt{3}$ N

$F = \mu R = \frac{1}{2}R$, so $R = 100\sqrt{3}$ N

R($\downarrow$): $mg + 100\sin 30° - R = 0$

$$m = \frac{100\sqrt{3} - 100\sin 30°}{g}$$

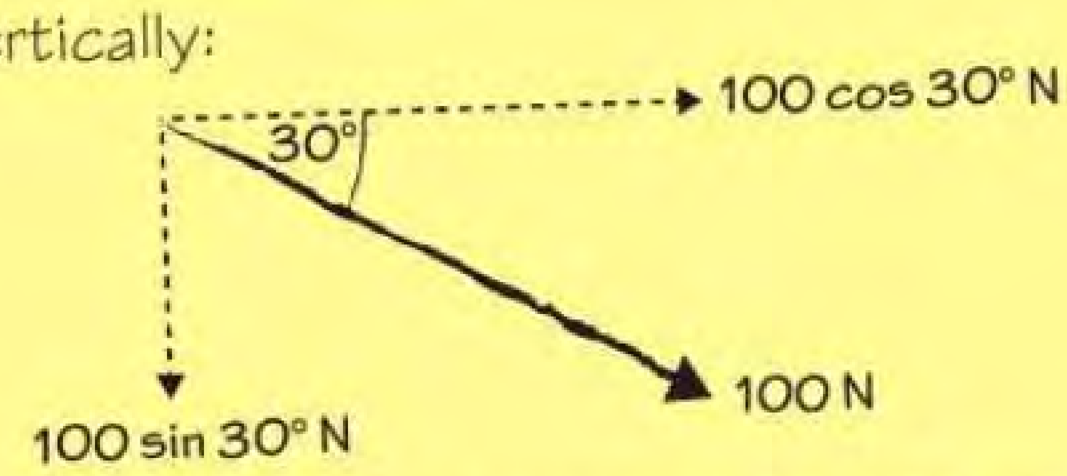
$= 12.571... = 13$ kg (2 s.f.)

Problem solved!

Don't confuse **constant speed** with constant acceleration. Constant acceleration requires a resultant force. If an object is moving with constant speed, the resultant force on it is **zero**.

You will need to use problem-solving skills throughout your exam – **be prepared!**

You need to resolve the 100 N force horizontally and vertically:



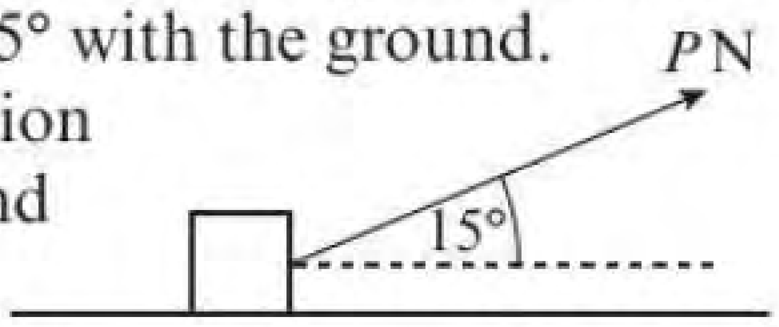
Use the **horizontal** component to work out the size of the frictional force, and then use this to work out the normal reaction, R.

Then use the **vertical** component to work out the mass of the box, m.

Remember that there are **two** forces acting vertically downwards on the block – its weight (mg) and the vertical component of the 100 N force.

Now try this

A block of mass 20 kg is being pulled along rough horizontal ground at a constant speed using a rope. The tension in the rope is P N and it makes an angle of 15° with the ground. The coefficient of friction between the ground and the block is 0.5.



Resolve vertically and horizontally to find two **simultaneous equations** in P and the normal reaction, R.

(a) Find the value of P. **(8 marks)**

Answer

The rope is cut.

(b) Explain whether the normal reaction between the ground and the block will increase or decrease. **(2 marks)**

Answer

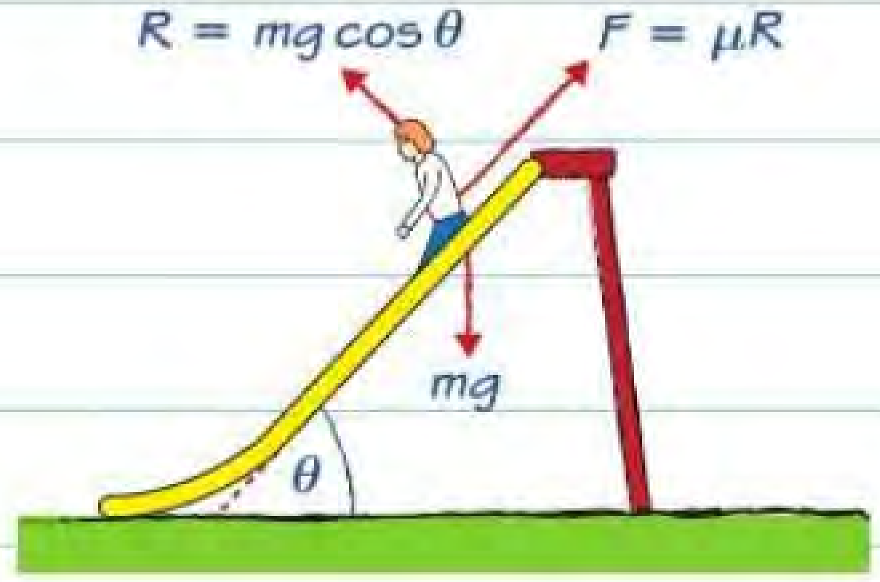
Sloping planes

Questions involving **friction** and sloping (or **inclined**) planes are really common. For a reminder about **resolving forces** parallel and perpendicular to a sloping plane have a look at page 167.

An object moving freely on a rough plane inclined at an angle θ to the horizontal has **three** forces acting on it:

- **weight** $= mg$
- **normal reaction**, $R = mg\cos\theta$
- **friction**, $F = \mu R$.

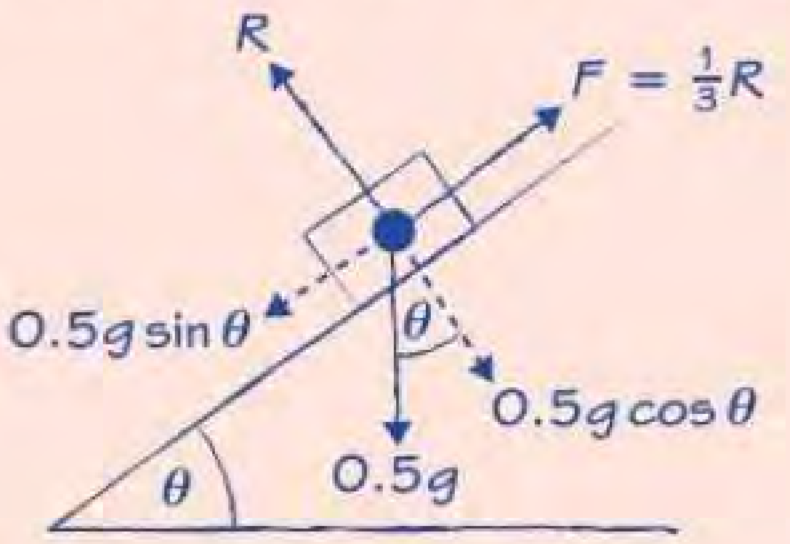
If the component of the weight that acts down the slope ($mg\sin\theta$) is greater than the friction, the object will **accelerate**.



Worked example

A small brick of mass 0.5 kg is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$, and released from rest. The coefficient of friction between the brick and the plane is $\frac{1}{3}$

Find the acceleration of the brick. **(9 marks)**



R(↖): $R - 0.5g\cos\theta = 0$

$R = 0.5g\cos\theta$

Using $\cos\theta = \frac{3}{5}$: $R = 0.3g$

R(↙): $0.5g\sin\theta - F = 0.5a$

$0.5g\sin\theta - \frac{1}{3}R = 0.5a$

Substituting the value of R into this equation:

$0.5g\sin\theta - \frac{1}{3} \times 0.3g = 0.5a$

Using $\sin\theta = \frac{4}{5}$: $0.4g - 0.1g = 0.5a$

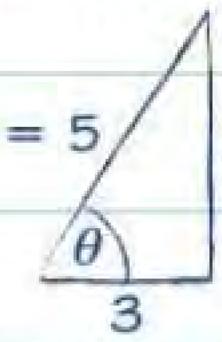
$0.3g = 0.5a$

$a = \frac{3g}{5} = 5.9\,\text{m s}^{-2}$ (2 s.f.)

Trig ratios

If you are given one trigonometric ratio you can work out the values of the other two by sketching a triangle. Here are two examples:

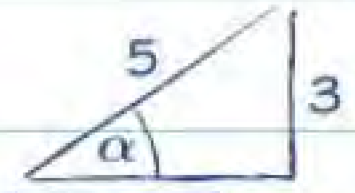
1 $\tan\theta = \frac{4}{3}$ $\sqrt{3^2 + 4^2} = 5$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

2 $\sin\alpha = \frac{3}{5}$



$\sqrt{5^2 - 3^2} = 4$

$\cos\alpha = \frac{4}{5}$

$\tan\alpha = \frac{3}{4}$

Ratios involving 3-4-5 triangles like these are common. They are designed to make the question **easier** to answer, so don't work out $\tan^{-1}\left(\frac{4}{3}\right)$ or $\sin^{-1}\left(\frac{3}{5}\right)$ to get the angle as a decimal.

Friction depends on the **normal reaction** so it's usually a good idea to resolve **perpendicular** to the plane first. This gives you the magnitude of the normal reaction, which you can use to find the frictional force. Subtract the friction from the component of the weight that acts **down** the plane to find the resultant force down the plane acting on the brick.

There is more about resultant forces and $F = ma$ on page 153.

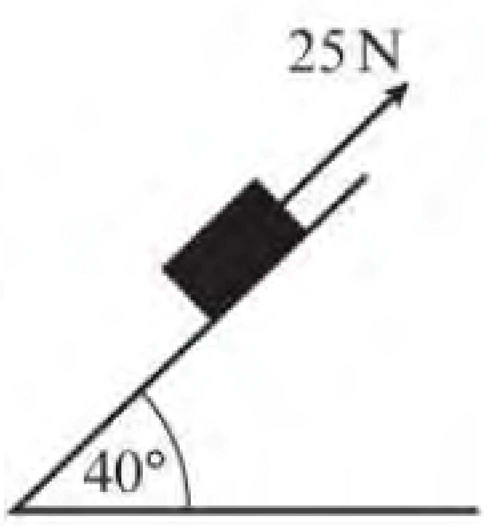
Now try this

A box of mass 3 kg is pulled up a rough plane by a force of magnitude 25 N acting parallel to the line of greatest slope of the plane. The plane is inclined at an angle of 40° to the horizontal. The coefficient of friction between the box and the plane is 0.2

By modelling the box as a particle, find

(a) the normal reaction of the plane on the box **Answer** **(3 marks)**

(b) the acceleration of the box. **Answer** **(5 marks)**



Limiting equilibrium

If a particle on a **rough surface** is in limiting equilibrium (or **on the point of** moving), then

- it is **static** (so the resultant of all the forces acting on it is **zero**)

Have a look at page 172 for more on dealing with static particles.

- the frictional force is $F = \mu R$.

Look at page 168 for a reminder about friction.

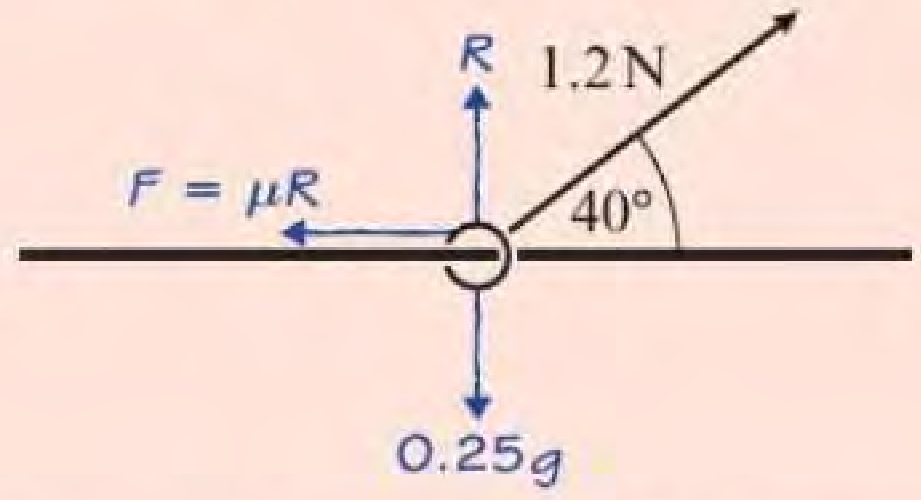
Add the weight of the ring, the normal reaction and the frictional force to the diagram. The ring is in **limiting equilibrium** so you know that

- the resultant force in any direction will be 0
- $F = \mu R$.

Resolve vertically to find the normal reaction, then resolve horizontally to find the value of μ.

Remember to use **unrounded** values in later calculations. In this question, you could lose marks by using $R = 1.7$ N in part (b). Use the Ans button on your calculator to enter the exact value, or use at least 4 decimal places. You have used $g = 9.8\,\text{m s}^{-2}$ in this calculation, so you need to round any final answers to **2 significant figures**.

Worked example



A small ring of mass 0.25 kg is threaded on a fixed rough horizontal rod. The ring is pulled upwards by a light string which makes an angle of 40° with the horizontal, as shown. The string and the rod are in the same vertical plane. The tension in the string is 1.2 N and the coefficient of friction between the ring and the rod is μ.

Given that the ring is in limiting equilibrium, find

(a) the normal reaction between the ring and the rod **(4 marks)**

$R(\uparrow)$: $R + 1.2\sin 40° - 0.25g = 0$

$R = 0.25g - 1.2\sin 40°$

$= 1.6786... = 1.7\text{ N (2 s.f.)}$

(b) the value of μ. **(6 marks)**

$R(\rightarrow)$: $1.2\cos 40° - F = 0$

$\mu R = 1.2\cos 40°$

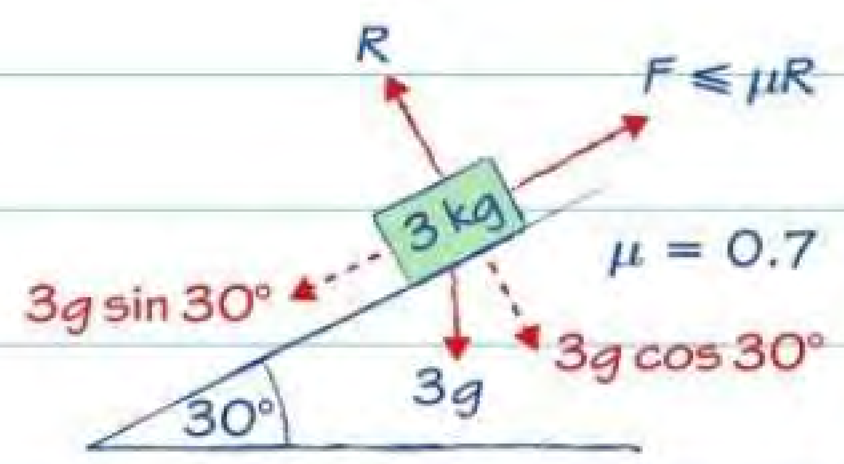
$\mu = \dfrac{1.2\cos 40°}{1.6786...}$

$= 0.5476... = 0.55\text{ (2 s.f.)}$

$F \leqslant \mu R$

If an object on a rough plane is **not moving**, the value of F is **only** as large as it needs to be to resist motion. You sometimes write $F_{MAX} = \mu R$.

This block is resting on a sloping plane. The component of its weight acting down the slope is $3g\sin 30° = 14.7$ N. This is less than $\mu R = 2.1g\cos 30° = 18$ N, so the block does not move. It is in equilibrium, with $F = 14.7$ N.



Now try this

Check whether the particle is on the point of moving **up** or **down** the slope. Here it is on the point of moving **up** the slope, so the frictional force will act **down** the slope.

A particle P of mass 5 kg is held at rest on a rough plane.
The plane is inclined to the horizontal at an angle α where $\cos\alpha = \frac{4}{5}$
The coefficient of friction between the particle and the plane is 0.5
The particle is held in place by a horizontal force of magnitude X N.
The particle is in equilibrium and on the point of moving up the plane.
Find the value of X. Answer **(7 marks)**