Summary of key points

1 The event A and B can be written as $A \cap B$. The '$\cap$' symbol is the symbol for **intersection**.

The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for **union**.

The event not A can be written as A'. This is also called the **complement** of A.

2 The probability that B occurs given that A has already occurred is written as $P(B|A)$.

For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4 $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ so $P(B \cap A) = P(B|A) \times P(A)$

Conditional probability

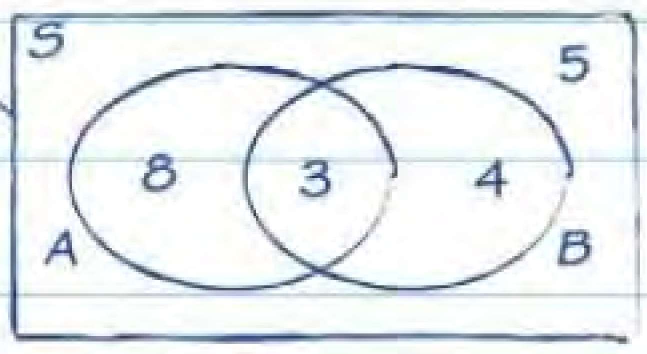
If one event **has already occurred**, the probability of other events occurring might **change**. This is called conditional probability. The probability that an event *X* occurs **given that** an event *Y* has **already** occurred is written as P(*X* | *Y*).

Using Venn diagrams

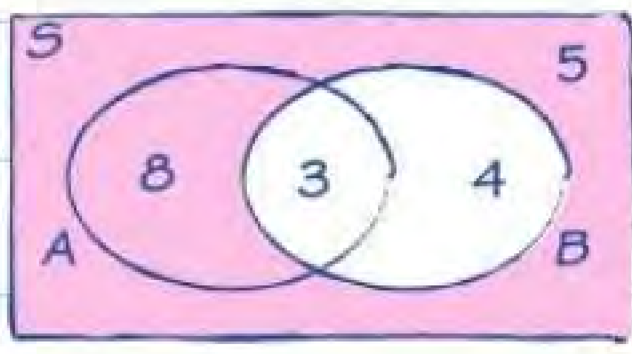
You can solve some conditional probability problems using a Venn diagram. If an event has already occurred, then the sample space for the other events is **restricted**. These Venn diagrams show the outcomes of two events, *A* and *B*:

Complete sample space

$$P(A) = \frac{8+3}{8+3+4+5} = \frac{11}{20}$$



Event *B* occurs

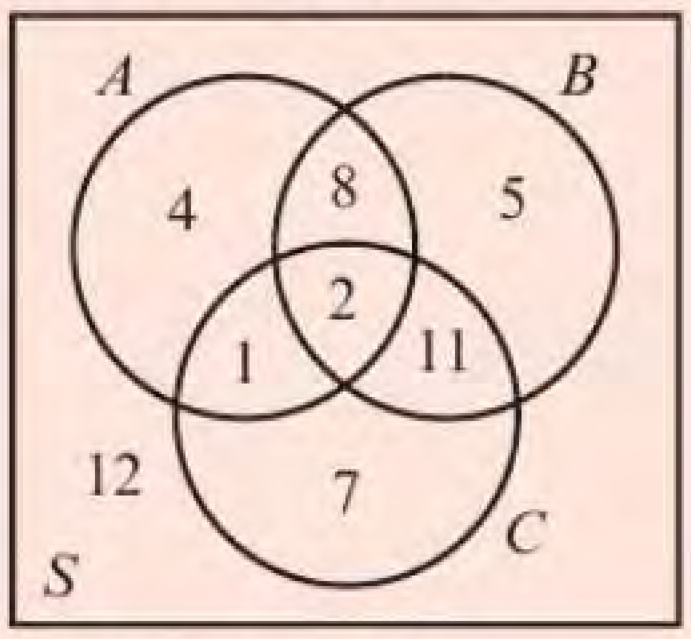


Restricted sample space **given that** event *B* has occurred

$$P(A \mid B) = \frac{3}{3+4} = \frac{3}{7}$$

Worked example

This Venn diagram shows the burger toppings chosen by a group of 50 diners at a restaurant. The choices are avocado, bacon and cheese.



A diner is selected at random.

(a) Given that the diner chooses bacon, find the probability that she also chooses avocado. **(2 marks)**

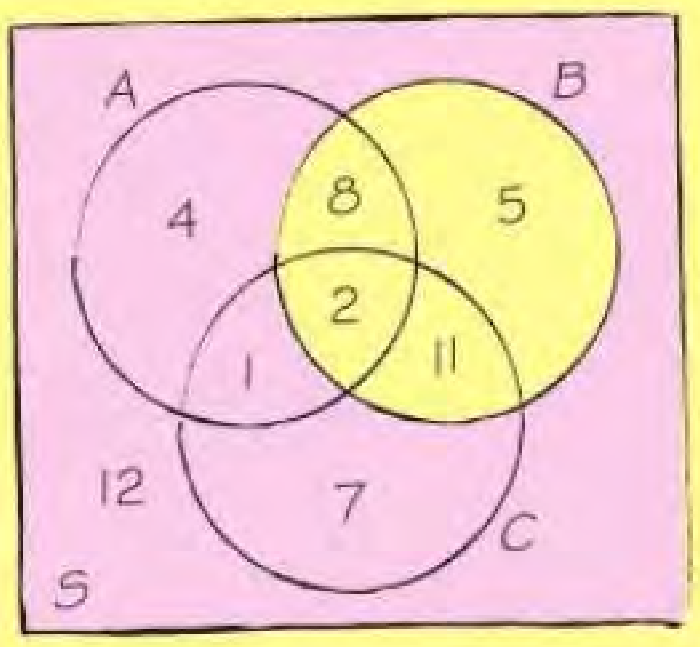
$$\frac{8+2}{8+2+11+5} = \frac{10}{26} = \frac{5}{13}$$

A second diner is selected at random.

(b) Given that the diner chooses at least one of the three toppings, find the probability that she chooses all three. **(3 marks)**

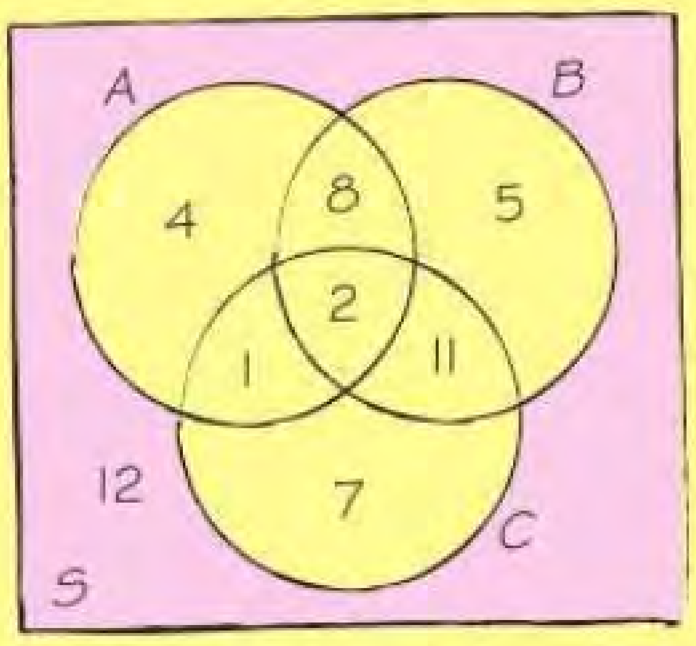
$$\frac{2}{4+8+5+1+2+11+7} = \frac{2}{38} = \frac{1}{19}$$

(a) Look at this restricted sample space:

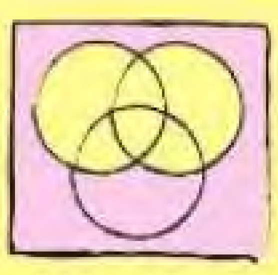


Of the 8 + 2 + 11 + 5 = 26 diners who chose bacon, 8 + 2 = 10 also chose avocado.

(b) Look at this restricted sample space:



Look at this restricted sample space:



Now try this

Look at the Venn diagram in the Worked example. A third diner is selected at random. Given that the diner chooses either avocado or bacon, work out the probability that she chooses cheese.

Answer **(3 marks)**

Had a look ☐ Nearly there ☐ Nailed it! ☐

Probability formulae

These two probability formulae are given in the formulae booklet. If a probability problem involves **set notation** (∪ and ∩) and it does not ask you to draw a Venn diagram, it might be easier to solve it using these formulae:

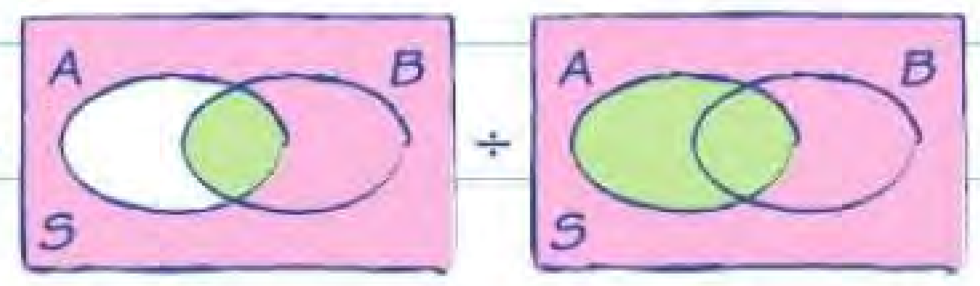
Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A)$

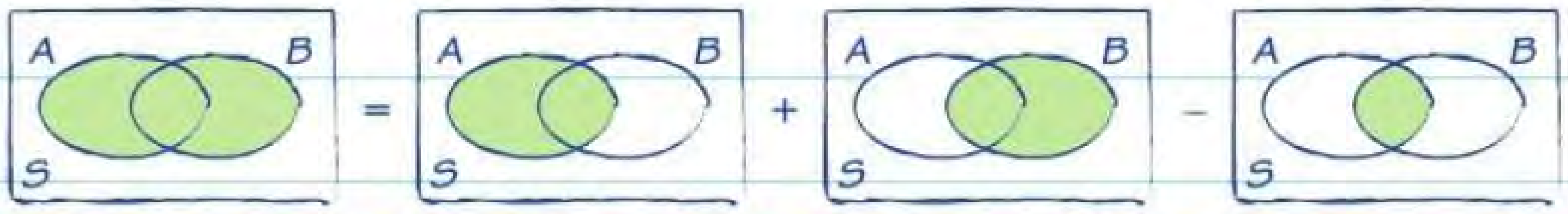
This is the **conditional probability rule**.
Look at the restricted sample space **after** event A has occurred:

$P(B \mid A) =$



$= \frac{P(A \cap B)}{P(A)}$

This is sometimes called the **addition rule**.



Worked example

Two events A and B are such that
$P(A \cap B') = 0.15$, $P(B) = 0.3$ and $P(A \mid B) = 0.2$
Find the value of

(a) $P(A \cap B)$ **(2 marks)**

$P(A \cap B) = P(B) \times P(A \mid B)$
$= 0.3 \times 0.2$
$= 0.06$

(b) $P(A \cup B)$ **(1 mark)**

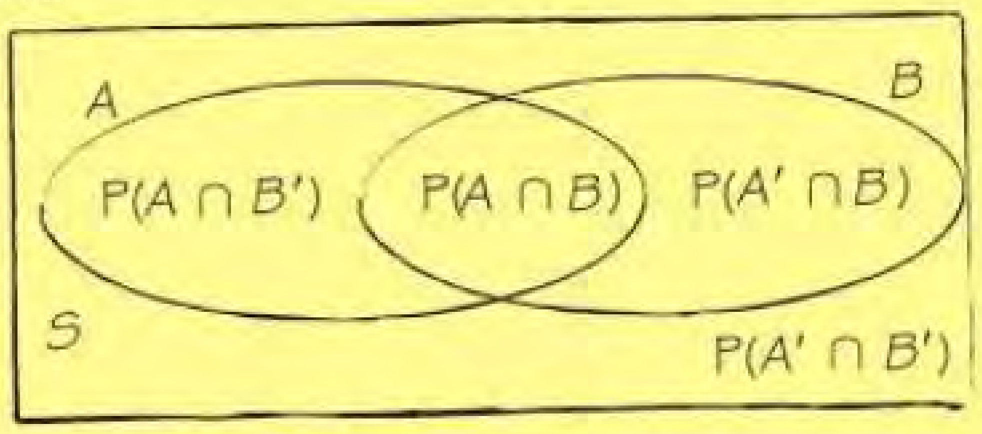
$0.15 + 0.3 = 0.45$

(c) $P(A)$ **(2 marks)**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.45 = P(A) + 0.3 - 0.06$
$P(A) = 0.21$

(a) This is the conditional probability rule.

(b) Use the fact that
$P(A \cup B) = P(A \cap B') + P(B)$
You can see this from this Venn diagram, but it is a useful formula to learn.



(c) This is the addition rule. You could also use the fact that
$P(A) = P(A \cap B') + P(A \cap B)$
$= 0.15 + 0.06$

Now try this

In a survey of a group of students $\frac{2}{3}$ played competitive sports and $\frac{2}{5}$ played video games. Of those who played competitive sports, $\frac{7}{25}$ also played video games.

Find the probability that a randomly selected member of the group

(a) plays competitive sports and video games **(2 marks)**

Answer

(b) plays neither competitive sports nor video games. **(4 marks)**

Answer

You might need to define your own events. For the randomly selected student, there are two events:
C = plays competitive sports
V = plays video games.
You can rewrite the information given as
$P(C) = \frac{2}{3}$, $P(V) = \frac{2}{5}$ and $P(V \mid C) = \frac{7}{25}$

(a) You want to find $P(C \cap V)$. You can use the formula $P(C \cap V) = P(C) \times P(V \mid C)$

(b) You want to find $P(C' \cap V')$.
Use your result to part (a), and the addition rule.
Then use the fact that
$P(C' \cap V') = 1 - P(C \cup V)$