# Laws of Indices and Surds

*You use the laws of indices all the time in maths — when you're integrating, differentiating and ...er... well loads of other places. So take the time to get them sorted now.*

## *Three mega-important **Laws of Indices***

You **must** know these three rules. I can't make it any clearer than that.

$$a^m \times a^n = a^{m+n}$$

If you **multiply** two numbers — you **add** their powers.

$a^2a^3 = a^5$

$x^{-2}x^5 = x^3$

$p^{\frac{1}{2}} \cdot p^{\frac{1}{4}} = p^{\frac{3}{4}}$ (The dot just means 'multiplied by'.)

$(a+b)^2(a+b)^5 = (a+b)^7$

$y \cdot y^3 = y^4$ (Since $y = y^1$.)

$ab^3 \cdot a^2b = a^3b^4$ (Add the powers of a and b separately.)

$$\frac{a^m}{a^n} = a^{m-n}$$

If you **divide** two numbers — you **subtract** their powers.

$\frac{x^5}{x^2} = x^3$

$\frac{x^{\frac{3}{4}}}{x} = x^{-\frac{1}{4}}$

$\frac{x^3y^2}{xy^3} = x^2y^{-1}$ (Subtract the powers of x and y separately.)

$$(a^m)^n = a^{mn}$$

If you have a **power** to the **power of something else** — **multiply** the powers together.

$(x^2)^3 = x^6$

$\{(a+b)^3\}^4 = (a+b)^{12}$

$(p^3)^{-2} = p^{-6}$

$(ab^2)^4 = a^4(b^2)^4 = a^4b^8$ (This power applies to both bits inside the brackets.)

## *Other important stuff about **Indices***

You can't get very far without knowing this sort of stuff. Learn it — you'll definitely be able to use it.

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

You can write **roots** as powers...

$x^{\frac{1}{5}} = \sqrt[5]{x}$

$4^{\frac{1}{2}} = \sqrt{4} = 2$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

A power that's a **fraction** like this is the **root of a power** — or the **power of a root**.

$9^{\frac{3}{2}} = (9^{\frac{1}{2}})^3 = (\sqrt{9})^3 = 3^3 = 27$

$81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3 = (\sqrt[4]{81})^3 = 3^3 = 27$

(It's often easier to work out the root first, then raise it to the power.)

$$a^{-m} = \frac{1}{a^m}$$

A **negative** power means it's on the bottom line of a fraction.

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$(x+1)^{-1} = \frac{1}{x+1}$

$$a^0 = 1$$

This is true for **any** number or letter.

$x^0 = 1$

$2^0 = 1$

$(a+b)^0 = 1$

## ***Surds** are sometimes the only way to give an **Exact Answer***

If you put $\sqrt{2}$ into a calculator, you'll get something like 1.414213562...
But if you square 1.414213562, then you get 1.999999999.
And no matter how many decimal places you use, you'll never get **exactly** 2.

To write the exact, spot on value you can **use surds**.
There are three **rules** you'll need to know to be able to use surds properly.

**Rules of Surds**

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$a = (\sqrt{a})^2 = \sqrt{a}\sqrt{a}$

# Laws of Indices and Surds

## *Use the **Three Rules** to deal with **Surds***

**Examples:** a) Simplify (i) $\sqrt{12}$ (ii) $\sqrt{\frac{3}{16}}$ b) Find $(2\sqrt{5}+3\sqrt{6})^2$

a) To **simplify** a surd, make the number in the $\sqrt{\ }$ sign **smaller**, or get rid of a **fraction** in the $\sqrt{\ }$ sign.

(i) $\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = \mathbf{2\sqrt{3}}$ — Using $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

(ii) $\sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{\sqrt{16}} = \mathbf{\frac{\sqrt{3}}{4}}$ — Using $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

b) Multiply surds very **carefully** — it's easy to make a silly mistake.

$$(2\sqrt{5}+3\sqrt{6})^2 = (2\sqrt{5}+3\sqrt{6})(2\sqrt{5}+3\sqrt{6})$$
$$= (2\sqrt{5})^2 + (2\times 2\sqrt{5}\times 3\sqrt{6}) + (3\sqrt{6})^2$$
$$= (2^2\times\sqrt{5}^2) + (2\times 2\times 3\times\sqrt{5}\times\sqrt{6}) + (3^2\times\sqrt{6}^2)$$
$$= 20 + 12\sqrt{30} + 54$$
$$= \mathbf{74 + 12\sqrt{30}}$$

$= 4\times 5 = 20$; $= 12\sqrt{5}\sqrt{6} = 12\sqrt{30}$; $= 9\times 6 = 54$

## *You might need to **Rationalise the Denominator***

**Rationalising the denominator** means getting rid of the surds from the bottom of a fraction.

**Example:** Show that $\frac{9}{\sqrt{3}} = 3\sqrt{3}$

Multiply the top and bottom by the denominator.

$$\frac{9}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$
$$= \frac{9\sqrt{3}}{3}$$
$$= \mathbf{3\sqrt{3}}$$

**Example:** Rationalise the denominator of $\frac{1}{1+\sqrt{2}}$

Multiply the top and bottom by the denominator (but change the sign in front of the surd).

$$\frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}$$
$$= \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{1-\sqrt{2}}{1^2-\sqrt{2}+\sqrt{2}-\sqrt{2}^2}$$
$$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \mathbf{-1+\sqrt{2}}$$

This works because: $(a+b)(a-b) = a^2 - b^2$

## *Practice Questions*

Q1 Simplify these:

a) $x^3\cdot x^5$ b) $a^7\cdot a^8$ c) $\frac{x^8}{x^2}$ d) $(a^2)^4$ e) $(xy^2)\cdot(x^3yz)$ f) $\frac{a^2b^4c^6}{a^3b^2c}$

Bruce lived by the sword and died by the surd.

Q2 Work out the following: a) $16^{\frac{1}{2}}$ b) $8^{\frac{1}{3}}$ c) $16^{\frac{3}{4}}$ d) $x^0$ e) $49^{-\frac{1}{2}}$

Q3 Simplify: a) $\sqrt{28}$ b) $\sqrt{\frac{5}{36}}$ c) $\sqrt{18}$ d) $\sqrt{\frac{9}{16}}$

Q4 Find $(6\sqrt{3}+2\sqrt{7})^2$

Q5 Rationalise the denominator of: $\frac{2}{3+\sqrt{7}}$

**Exam Questions**

Q1 Simplify a) $(5\sqrt{3})^2$ [1 mark]

b) $(5+\sqrt{6})(2-\sqrt{6})$ [2 marks]

Q2 Given that $10000\sqrt{10} = 10^k$, find the value of $k$. [2 marks]

Q3 Express $\frac{5+\sqrt{5}}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$, where $a$ and $b$ are integers. [4 marks]

## *Where does Poseidon keep his powers — indices...*

*For lots of these questions you can check your answers on your calculator. If you ever forget the rules of surds you can even write $\sqrt{\ }$ as $^{\frac{1}{2}}$ and manipulate the indices instead — e.g. $\sqrt{ab} = (ab)^{\frac{1}{2}} = a^{\frac{1}{2}}b^{\frac{1}{2}} = \sqrt{a}\sqrt{b}$. Very sneaky.*