Exponential *equations can be* ***Reduced to Linear Form***

Equations like $y = ax^n$ and $y = ab^x$ can be a bit awkward to use. Fortunately, using the **laws of logs**, they can be rewritten to look like a form you've seen before — good old $y = mx + c$. Just take **logs** of both sides and rearrange:

$$y = ax^n \Rightarrow \log y = n \log x + \log a$$

$$y = ab^x \Rightarrow \log y = x \log b + \log a$$

The equations look pretty horrendous now, I'll admit. But look at them carefully — they're just a nasty-looking version of y = mx + c.

Once the equations are in this form you can draw their **straight line graphs** — you just need to **label** the axes **log** x (top) or x (bottom) against **log** y. Now your graph is **easier** to work with than the exponential graph.

Using Exponentials and Logs

Example: The number of employees, p, working for a company t years after it was founded can be modelled by the equation $p = at^b$. The table below shows the number of employees the company has:

Age of company (t years)	2	5	8	13	25
Number of employees (p)	3	7	10	16	29

a) Show that $p = at^b$ can be written in the form $\log p = b \log t + \log a$.

b) Plot a graph of $\log t$ against $\log p$ and draw a line of best fit for your graph.

c) Use your graph to estimate the values of a and b in the equation $p = at^b$.

Starting with $p = at^b$, take logs of both sides:

a) $\log p = \log at^b$

Now use the laws of logs to rearrange into required form:

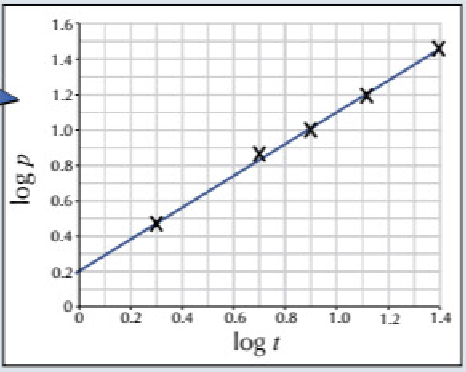
$\log p = \log a + \log t^b = b \log t + \log a$

Make a table of the values of log t and log p using p and t as given in the question:

b)

$\log t$	0.301	0.699	0.903	1.114	1.398
$\log p$	0.477	0.845	1.000	1.204	1.462

Now plot a graph of log t against log p and draw a line of best fit:



c) From part a), the graph has equation $\log p = b \log t + \log a$.
Compare this to $y = mx + c$:
b is the gradient of the line and
$\log a$ is the vertical intercept of the line.

Use the coordinates of two points on the line to find the gradient:
E.g. use coordinates (1.0, 1.1) and (0, 0.2):

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.1 - 0.2}{1.0 - 0} = \mathbf{0.9}$$

You can also read the vertical intercept off the graph — 0.2.
BUT this value is equal to log a, so take exponentials of both sides:
$a = 10^{0.2} = \mathbf{1.585 \text{ to } 3 \text{ d.p.}}$

So $b = 0.9$ and $a = 1.585$, and the original equation $p = at^b$ is $p = 1.585t^{0.9}$.

Be careful when using models like this to predict values outside the range of the given data — this is extrapolation (see p.183).

Practice Questions

Q1 If $6^{(3x+2)} = 9$, find x to 3 significant figures.

Q2 If $3^{(y^2-4)} = 7^{(y+2)}$, find y to 3 significant figures.

Q3 The value of a painting is modelled as increasing by 5% each year. If the initial price of the painting is £1000, find the time taken in years for the price to reach £2000. Give your answer to 1 d.p.

Exam Question

Q1 The yearly income from book sales of a particular author has tended to increase with time.
The table below shows his income from book sales over the first five years after his book was published.

Number of years after book published (t)	1	2	3	4	5
Income (£p thousand)	10	13	17	24	35

The relationship is modelled by the equation $p = ab^t$, where a and b are constants to be found.

a) Plot a graph of t against $\log_{10} p$. Draw, by eye, a line of best fit for your graph. [2 marks]

b) State, in terms of a and b, the gradient and vertical-axis intercept of your graph.
Hence use your graph to find the values of a and b. [4 marks]

c) Predict the author's income 10 years after his book was published. [1 mark]

d) Suggest one reason why the prediction in part c) might not be accurate. [1 mark]

Reducing to linear form is hard work — you'll sleep like a log tonight...

The results you get with the method shown in the example will depend on your line of best fit, so make sure you draw it carefully. It doesn't hurt to check your final answer using the values in the original table to see how well it fits the data. And don't forget that the vertical intercept is a log — you'll need to take exponentials to get the value you want.