Differentiation

Differentiation is a great way to work out gradients of graphs. You take a function, differentiate it, and you can quickly tell how steep a graph is. It's magic. No, wait, the other thing — it's calculus.

*Use this **Formula** to **Differentiate Powers of x***

'Derivative' just means 'the thing you get when you differentiate something'.

For a function $f(x) = x^n$, the **derivative** $f'(x)$ can be found using this formula:

$\frac{d}{dx}$ just means 'the derivative of the thing in the brackets with respect to x'.

$$f'(x) = \frac{d}{dx}(x^n) = nx^{n-1}$$

If you have y = (some function of x), its derivative is written $\frac{dy}{dx}$, which means 'the rate of change of y with respect to x'.

Functions are much easier to **differentiate** when they're written as **powers of x** — like writing $\sqrt{x}$ as $x^{\frac{1}{2}}$ (see p.6). When you've done this, you can use the formula in the box above to differentiate the function.

Use the differentiation formula:

For '**normal**' powers:

E.g. $f(x) = x^2$ ← n is just the power of x.

Here, $n = 2$, so:

$f'(x) = nx^{n-1} = 2x^1 = \mathbf{2x}$

For **negative** powers:

E.g. $f(x) = \frac{1}{x^2} = x^{-2}$ ← Always rewrite the function as a power of x.

Here $n = -2$, so:

$f'(x) = nx^{n-1} = -2x^{-3} = -\frac{2}{x^3}$

For **fractional** powers:

E.g. $f(x) = \sqrt{x} = x^{\frac{1}{2}}$. ← Write the square root as a power of x.

Here, $n = \frac{1}{2}$, so:

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

*Differentiate each term **Separately***

Even if there are loads of terms in the function, it doesn't matter.
Differentiate each bit **separately** and you'll be fine. Here are a couple of examples:

Example: Differentiate $y = 3\sqrt{x} = 3x^{\frac{1}{2}}$

If the function is being multiplied by a constant (3 in this case)...

...multiply the derivative by the same number.

$$\frac{dy}{dx} = 3\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$\frac{dy}{dx} = \frac{3}{2} \times x^{-\frac{1}{2}} = \frac{3}{2\sqrt{x}}$$

Example: Differentiate $y = 6x^2 + \frac{4}{\sqrt[3]{x}} - \frac{2}{x^2} + 1$

Write each term as a power of x, including the constants: $1 = x^0$

$$y = 6x^2 + 4x^{-\frac{1}{3}} - 2x^{-2} + x^0$$

Differentiate each bit separately and add or subtract the results.

$$\frac{dy}{dx} = 6(2x) + 4\left(-\frac{1}{3}x^{-\frac{4}{3}}\right) - 2(-2x^{-3}) + 0x^{-1}$$

$$\frac{dy}{dx} = 12x - \frac{4}{3\sqrt[3]{x^4}} + \frac{4}{x^3}$$

A constant always differentiates to 0.

*You can **Differentiate** to find **Gradients***

Differentiating tells you the **gradient** of a curve at any given point, which is the same as the gradient of the **tangent** to the curve at that point. Tangents will become a lot more important on the next page, so stay tuned...

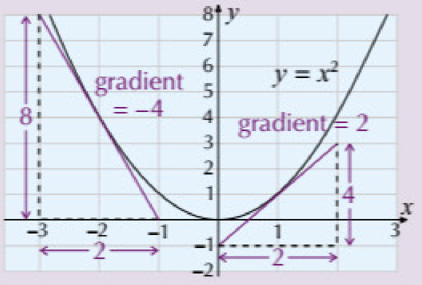
Example: Find the gradient of the graph $y = x^2$ at $x = 1$ and $x = -2$.

You need the **gradient** of the graph of: $y = x^2$

So **differentiate** this function to get: $\frac{dy}{dx} = 2x$

When $x = 1$, $\frac{dy}{dx} = 2(1) = 2$, so the gradient at $x = 1$ is **2**.

When $x = -2$, $\frac{dy}{dx} = 2(-2) = -4$, so the gradient at $x = -2$ is **−4**.



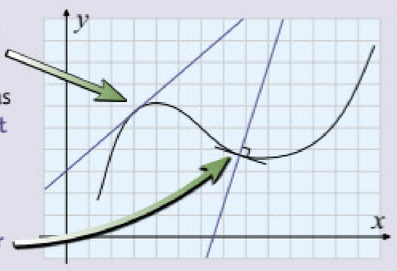
Differentiation

You can find the *Equation* of a *Tangent* or a *Normal* to a curve

Tangents and Normals

A **tangent** just touches the curve but doesn't go through it — it has the **same gradient** as the curve.

A **normal** is a line **perpendicular** (at right angles) to a curve.



There's more info on parallel and perpendicular lines on p.37.

Finding Tangents and Normals

1) Differentiate the function.
2) Find the gradient of the tangent or normal.
   For a **tangent**, this is the gradient of the curve.
   For a **normal**, this is $\dfrac{-1}{\text{gradient of the curve}}$.
3) Write the equation of the tangent or normal in the form $y - y_1 = m(x - x_1)$ or $y = mx + c$.
4) Use the coordinates of a point on the line to complete the equation of the line .

Tangents have the *Same Gradient* as the curve

Example: Find the tangent to the curve $y = (4 - x)(x + 2)$ at the point (2, 8).

To find the **gradient** of the curve (and the tangent), first write the equation in a **form** you can differentiate:

$$y = (4 - x)(x + 2) = 8 + 2x - x^2$$

Then **differentiate** it: $\dfrac{dy}{dx} = 2 - 2x$

The **gradient** of the tangent at (2, 8) will be the gradient of the curve at $x = 2$.

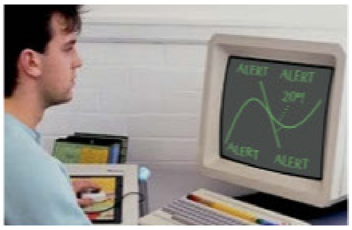
$$\text{At } x = 2,\ \frac{dy}{dx} = -2$$

So the tangent has equation $y - y_1 = -2(x - x_1)$, ← You could also use $y = mx + c$ here.

and since it passes through the point (2, 8), this becomes:

$$y - 8 = -2(x - 2) \quad \text{or} \quad y = 12 - 2x$$

You can give your answer in any of the forms from p.36.



"Sir, I'm picking up something abnormal in the system..."

Normals are *Perpendicular* to the curve

Example: Find the normal to the curve $y = \dfrac{(x+2)(x+4)}{6\sqrt{x}}$ at the point (4, 4).

Write the equation of the curve in a **form** you can differentiate:

$$y = \frac{x^2 + 6x + 8}{6x^{\frac{1}{2}}} = \frac{1}{6}x^{\frac{3}{2}} + x^{\frac{1}{2}} + \frac{4}{3}x^{-\frac{1}{2}}$$

← Divide everything on the top line by everything on the bottom line.

Then **differentiate** it:

$$\frac{dy}{dx} = \frac{1}{6}\left(\frac{3}{2}x^{\frac{1}{2}}\right) + \frac{1}{2}x^{-\frac{1}{2}} + \frac{4}{3}\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= \frac{1}{4}\sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{2}{3\sqrt{x^3}}$$

Find the **gradient** at (4, 4): At $x = 4$, $\dfrac{dy}{dx} = \dfrac{1}{4} \times 2 + \dfrac{1}{2 \times 2} - \dfrac{2}{3 \times 8} = \dfrac{2}{3}$

So the **gradient** of the **normal** is $-1 \div \dfrac{2}{3} = -\dfrac{3}{2}$ ← Because the gradients of perpendicular lines multiply to give –1.

And the **equation** of the normal is $y - y_1 = -\dfrac{3}{2}(x - x_1)$,

and since it passes through the point (4, 4), this becomes:

$$y - 4 = -\frac{3}{2}(x - 4) \quad \text{or} \quad 3x + 2y - 20 = 0$$

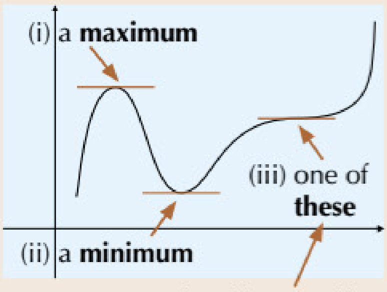
Stationary Points

Let me tell you about a special place — it's a magical point called a stationary point, where the gradient of the graph is zero, and there's a pot of g- oh no, wait, it's actually just the gradient thing. Turns out they're pretty ordinary, really.

Stationary Points** are when the gradient is **Zero

Stationary points are points on a graph where the curve **flattens out** — i.e. the **gradient** is **zero**.

A stationary point could be...



This kind of stationary point is called a 'point of inflection' — see p.90.

Example: Find the coordinates of the stationary points of the curve $y = 2x^3 - 3x^2 - 12x + 5$, and determine the nature of each.

You need to find where $\frac{dy}{dx} = 0$. So first, **differentiate** the function.

$$y = 2x^3 - 3x^2 - 12x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 6x - 12$$

Then set this derivative equal to **zero** and solve for x:

$$6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow (x + 1)(x - 2) = 0 \Rightarrow x = -1 \text{ or } x = 2$$

So the graph has **two** stationary points, at $x = -1$ and $x = 2$.

When $x = -1$, $y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5$
$= -2 - 3 + 12 + 5 = 12$

When $x = 2$, $y = 2(2)^3 - 3(2)^2 - 12(2) + 5$
$= 16 - 12 - 24 + 5 = -15$

So the stationary points are at **(–1, 12)** and **(2, –15)**. To be continued...

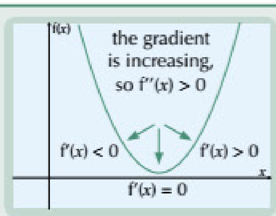
*Decide if it's a **Maximum** or a **Minimum** by differentiating **Again***

Once you've found where the stationary points are, you have to decide whether each one is a **maximum** or a **minimum** — that's all a question means when it says, '...determine the nature of the turning points' (a '**turning point**' is either a maximum or minimum — points of inflection don't count).

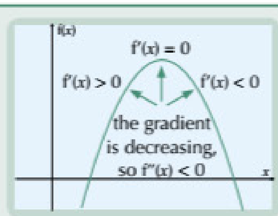
To decide whether a stationary point is a **maximum** or a **minimum**, differentiate again to find $\frac{d^2y}{dx^2}$, or $f''(x)$. $f''(x)$ is called the **second order derivative**, and is the **rate of change** of the gradient.

$\frac{d^2y}{dx^2}$ is read 'd 2 y by dx squared'.

If $f''(x) > 0$, it's a **minimum**.



If $f''(x) < 0$, it's a **maximum**.



If $\frac{d^2y}{dx^2} = 0$, then it could be any type of stationary point — see p.90.

Last time, on 'Strictly Come Differentiating'...

...you found that the stationary points were at **(–1, 12)** and **(2, –15)**.

$\frac{dy}{dx} = 6x^2 - 6x - 12$, so differentiate again: $\frac{d^2y}{dx^2} = 12x - 6$

Now find the value of $\frac{d^2y}{dx^2}$ at the stationary points:

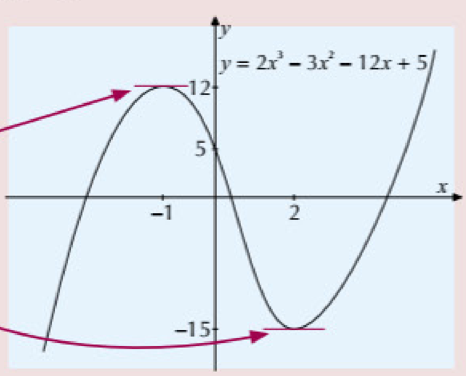
At $x = -1$, $\frac{d^2y}{dx^2} = 12(-1) - 6 = -18$

This is **negative**, so **(–1, 12) is a maximum**.

At $x = 2$, $\frac{d^2y}{dx^2} = 12(2) - 6 = 18$

This is **positive**, so **(2, –15) is a minimum**.

Finding the maximum and minimum points of a function makes it a lot easier to sketch the graph — check out the stuff on the next two pages.

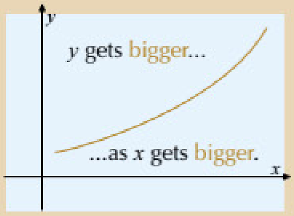


Stationary Points

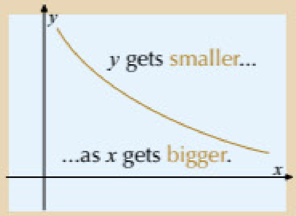
Find out if a function is *Increasing* or *Decreasing*

You can use differentiation to work out exactly where a function is **increasing** or **decreasing** — and how quickly.

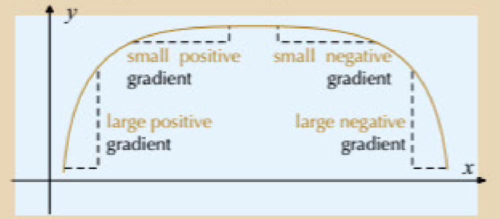
A function is **increasing** if the gradient is **positive**.



A function is **decreasing** if the gradient is **negative**.



The **bigger** the gradient, the **quicker** y changes as x changes.



Example: Find the range of x-values for which the graph of $f(x) = 4 + 3x - 2x^2$ is increasing.

You want to find where the gradient is positive, i.e. where $f'(x) > 0$. So differentiate:

$$f(x) = 4 + 3x - 2x^2 \Rightarrow f'(x) = 3 - 4x$$

Now find where $f'(x) > 0$: $\quad 3 - 4x > 0 \Rightarrow 4x < 3 \Rightarrow x < \frac{3}{4}$

Use differentiation to make *Curve Sketching* easier

Now, you might be wondering where I'm going with all of this stuff about increasing functions and stationary points and gradients... Well, as it happens, it's all really helpful for sketching graphs of complicated functions. So sit tight — here comes one mega-example that you won't soon forget:

1) Find where the curve crosses the *Axes*

Example: Sketch the graph of $f(x) = \frac{x^2}{2} - 2\sqrt{x}$ for $x \geq 0$.

The curve crosses the **y-axis** when $x = 0$ — so put $x = 0$ in the expression for y.

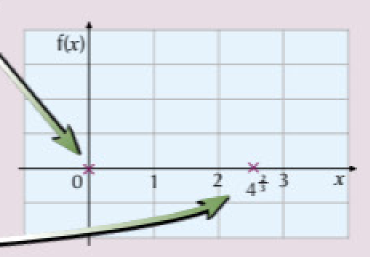
When $x = 0$, $f(x) = \frac{0^2}{2} - 2\sqrt{0} = 0$ — so the curve goes through the **origin**.

The curve crosses the **x-axis** when $f(x) = 0$. So solve:

$$\frac{x^2}{2} - 2\sqrt{x} = 0 \Rightarrow x^2 - 4x^{\frac{1}{2}} = 0 \Rightarrow x^{\frac{1}{2}}\left(x^{\frac{3}{2}} - 4\right) = 0$$

$$x^{\frac{1}{2}} = 0 \Rightarrow \mathbf{x = 0} \quad \text{and} \quad x^{\frac{3}{2}} - 4 = 0 \Rightarrow x^{\frac{3}{2}} = 4 \Rightarrow \mathbf{x = 4^{\frac{2}{3}}\ (\approx 2.5)}$$

So the curve crosses the x-axis when $x = 0$ and when $x \approx 2.5$.



2) *Differentiate* to find information about the *Gradient* and *Stationary Points*

Differentiating the function gives: $f'(x) = \frac{1}{2}(2x) - 2\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = x - x^{-\frac{1}{2}} = x - \frac{1}{\sqrt{x}}$

Find any **stationary points**: $\quad x - \frac{1}{\sqrt{x}} = 0 \Rightarrow x = \frac{1}{\sqrt{x}} \Rightarrow x^{\frac{3}{2}} = 1 \Rightarrow \mathbf{x = 1}$ ⟹ At $x = 1$, $f(x) = \frac{1}{2} - 2 = -\frac{3}{2}$

The gradient is **positive** when: $\quad x - \frac{1}{\sqrt{x}} > 0 \Rightarrow x > \frac{1}{\sqrt{x}} \Rightarrow x^{\frac{3}{2}} > 1 \Rightarrow \mathbf{x > 1}$

and it's **negative** when $x < 1$ — so the function is decreasing for $0 < x < 1$, then increasing for $x > 1$.
So you know that $\left(1, -\frac{3}{2}\right)$ **is a minimum**. You can check this by differentiating again:

$$f''(x) = 1 - \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) = 1 + \frac{1}{2\sqrt{x^3}} \qquad f''(1) = 1 + \frac{1}{2\sqrt{1^3}} = 1 + \frac{1}{2} > 0 \text{ so } \mathbf{x = 1 \text{ is a minimum.}}$$

Stationary Points

3) Find out what happens when x gets Big

You can also try and decide what happens as x gets very **big** — in both the positive and negative directions. There's a handy trick you can use to help with this when your function is made up of **powers of x** — **factorise** to take out the **highest power of x** from every term.

If the highest power of x is negative, then the graph will flatten out as x gets larger — see p.30 for more.

Factorise f(x) by taking the **biggest** power outside the brackets...

$$\frac{x^2}{2} - 2\sqrt{x} = x^2\left(\frac{1}{2} - 2x^{-\frac{3}{2}}\right) = x^2\left(\frac{1}{2} - \frac{2}{x^{\frac{3}{2}}}\right)$$

As x gets large, the $\frac{2}{x^{\frac{3}{2}}}$ gets smaller and smaller — so the bit in brackets gets closer to $\frac{1}{2}$.

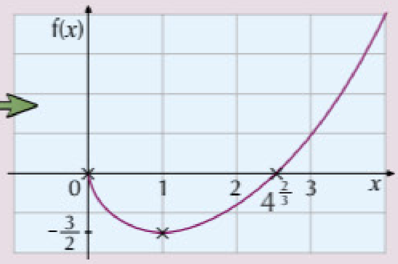
So as x gets larger, f(x) gets closer and closer to $\frac{1}{2}x^2$ — and this just keeps growing and growing.

So the final graph looks like this:

It passes through **(0, 0)** and **($4^{\frac{2}{3}}$, 0)**,

has a minimum point at $\left(1, -\frac{3}{2}\right)$,

and gets larger and larger as x increases.



You should normally also think about what happens when x is big and negative, but you don't have to here as the graph is for x ≥ 0.

Practice Questions

Q1 Find the stationary points of the graph of $y = x^3 - 6x^2 - 63x + 21$.

Q2 Find the stationary points of the function $y = x^3 + \frac{3}{x}$.
Decide whether each stationary point is a minimum or a maximum.

Q3 Find when these functions are increasing and decreasing:
a) $y = 6(x + 2)(x - 3)$ b) $y = \frac{1}{x^2}$

Q4 Sketch the graph of $y = x^3 - 4x$, clearly showing the coordinates of any turning points.

Exam Questions

Q1 a) Find $\frac{dy}{dx}$ for the curve $y = 6 + \frac{4x^3 - 15x^2 + 12x}{6}$. [2 marks]
b) Hence find the coordinates of the stationary points of the curve. [3 marks]
c) Determine the nature of each stationary point. [3 marks]

Q2 a) Find the coordinates of the stationary points for the curve $y = (x - 1)(3x^2 - 5x - 2)$. [4 marks]
b) Determine whether each of these points is a maximum or minimum. [3 marks]
c) Sketch the graph of $y = (x - 1)(3x^2 - 5x - 2)$. [3 marks]

Q3 The function $f(x) = \frac{1}{2}x^4 - 3x$ has a single stationary point.
a) Find the coordinates of the stationary point. [3 marks]
b) Determine the nature of the stationary point. [2 marks]
c) State the range of values of x for which f(x) is:
(i) increasing [1 mark]
(ii) decreasing [1 mark]
d) Sketch the graph of $y = f(x)$. [2 marks]

Curve sketching's important — but don't take my word for it...

Curve sketching — an underrated skill, in my opinion. As Shakespeare once wrote, 'Those who can do fab sketches of graphs and stuff are likely to get pretty good grades in maths exams, no word of a lie'. Well, he probably would've written something like that if he was into maths. And he would've written it because graphs are helpful when you're trying to work out what a question's all about — and once you know that, you can decide the best way forward. And if you don't believe me, remember the saying of the ancient Roman Emperor Julius Caesar, 'If in doubt, draw a graph'.

Using Differentiation

Differentiation isn't just mathematical daydreaming. It can be applied to real-life problems. For instance, you can use differentiation to find out the maximum possible volume of a box, given a limited amount of cardboard. Thrilling.

*Differentiation is used to find **Rates of Change** in **Mechanics**...*

Rates of change are particularly important in **mechanics** (you'll see more of this in Section 16) — and that means **differentiation** is important. You should know that **speed** is the rate of change of **distance travelled**, and **acceleration** is the rate of change of **speed** (see p.190 for more).

Example: The distance travelled by a car, s (in m), t seconds after it moves off, is given by: $s = \frac{9}{4}t^2 - \frac{1}{3}t^3$ for $0 \le t \le 4$. Find: a) the car's speed after 3 seconds, b) when the car is decelerating.

a) The **speed** of the car is the **rate of change** of the **distance travelled**, i.e. $\frac{ds}{dt}$:

$\frac{ds}{dt} = \frac{9}{2}t - t^2$ When $t = 3$, $\frac{ds}{dt} = \frac{9}{2}(3) - (3)^2 = 13.5 - 9 = $ **4.5 ms^{-1}**

The units of $\frac{ds}{dt}$ are $\frac{\text{units of s}}{\text{units of t}}$, which are $\frac{\text{m}}{\text{s}}$, or ms^{-1}.

b) If the car is **decelerating**, then its acceleration is **negative**.

Acceleration is the **rate of change** of **speed**, i.e. $\frac{d}{dt}\left(\frac{ds}{dt}\right)$ or $\frac{d^2s}{dt^2}$:

$\frac{d^2s}{dt^2} = \frac{9}{2} - 2t$ $\frac{d^2s}{dt^2} < 0 \Rightarrow \frac{9}{2} - 2t < 0 \Rightarrow 4t > 9 \Rightarrow t > 2.25 \text{ s}$

So the car is decelerating for **$2.25 < t \le 4$**.

Remember that the equation given is only valid up to t = 4.

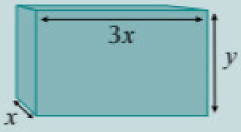
*...and for finding **Maximum** or **Minimum Values** for **Volume** and **Area***

To find the maximum for a shape's volume, all you need is an equation for the volume **in terms of only one variable** — then just **differentiate as normal**. But examiners don't hand it to you on a plate — there's usually one too many variables chucked in. So you need to know how to manipulate the information to get rid of that unwanted variable.

Example: A jewellery box with a lid and dimensions $3x$ cm by x cm by y cm is made using a total of 450 cm² of wood.

a) Show that the volume of the box can be expressed as: $V = \frac{675x - 9x^3}{4}$.

b) Use calculus to find the maximum possible volume.



a) You know the basic equation for **volume**: $V = \text{width} \times \text{depth} \times \text{height} = 3x \times x \times y = 3x^2y$

But the question asks for volume in terms of **x only** — you don't want that pesky y in there.
So you need to find y **in terms of x** and substitute that in.

Write an expression for the **surface area**:

$$A = 2[(3x \times x) + (3x \times y) + (x \times y)] = 450 \Rightarrow 3x^2 + 4xy = 225$$

Be careful when finding the surface area — here there's a lid so there are two of each side, but sometimes you'll get an open-topped shape.

Then rearrange to find an expression for y:

$$4xy = 225 - 3x^2 \Rightarrow y = \frac{225 - 3x^2}{4x}$$

Finally, **substitute** this into the equation for the volume: $V = 3x^2y = 3x^2 \times \frac{225 - 3x^2}{4x} = \frac{3x(225 - 3x^2)}{4}$

$$\Rightarrow V = \frac{675x - 9x^3}{4} \text{ as required}$$

b) You want to find the **maximum** value of V, so **differentiate** and set $\frac{dV}{dx} = 0$:

$\frac{dV}{dx} = \frac{675 - 27x^2}{4}$, $\frac{dV}{dx} = 0 \Rightarrow \frac{675 - 27x^2}{4} = 0 \Rightarrow 675 = 27x^2 \Rightarrow x^2 = 25 \Rightarrow$ **$x = 5$**

Ignore the other solution, $x = -5$, since you can't have a negative length.

(You could check that this is a maximum by finding $\frac{d^2V}{dx^2}$ when $x = 5$: $\frac{d^2V}{dx^2} = -\frac{27}{2}x = -\frac{135}{2} < 0$)

$\frac{d^2V}{dx^2}$ is negative — so it's a maximum.

So the maximum volume is: $V = \frac{(675 \times 5) - (9 \times 5^3)}{4} = $ **562.5 cm^3**

Using Differentiation

Another example? Coming right up. You want pie in this one? No probl- oh hang on — do you mean pie, or pi? I'll use both, just to make sure — I wouldn't want you to be disappointed. I really spoil you, don't I...

Example: Ned uses a circular tin to bake his pies in. The tin is t cm high with a d cm diameter. The volume of the pie tin is 1000 cm³.

a) Prove that the surface area of the tin, $A = \frac{\pi}{4}d^2 + \frac{4000}{d}$.

b) Find the minimum surface area.

a) A = area of tin's base + area of tin's curved face $= \pi\left(\frac{d}{2}\right)^2 + (\pi d \times t) = \frac{\pi}{4}d^2 + \pi dt$

This shape is open-topped, so only count the area of the circle once.

You want to get rid of the t, so use the given value of volume to find an expression for t in terms of d:

$$V = \pi\left(\frac{d}{2}\right)^2 t = 1000 \Rightarrow \pi d^2 t = 4000 \Rightarrow t = \frac{4000}{\pi d^2}$$

Substitute your expression for t into the equation for surface area:

$$A = \frac{\pi}{4}d^2 + \left(\pi d \times \frac{4000}{\pi d^2}\right) \Rightarrow \mathbf{A = \frac{\pi}{4}d^2 + \frac{4000}{d}} \text{ as required.}$$

b) Differentiate and find the stationary point:

$$\frac{dA}{dd} = \frac{\pi}{2}d - \frac{4000}{d^2} \Rightarrow \frac{\pi}{2}d - \frac{4000}{d^2} = 0 \Rightarrow d^3 = \frac{8000}{\pi} \Rightarrow d = \frac{20}{\sqrt[3]{\pi}}$$

Check it's a minimum: $\frac{d^2A}{dd^2} = \frac{\pi}{2} + \frac{8000}{d^3} = \frac{\pi}{2} + \frac{8000}{\left(\frac{8000}{\pi}\right)} = \frac{3\pi}{2}$

$\frac{d^2A}{dd^2}$ is positive — so it's a minimum.

Calculate the area for this value of d: $A = \frac{\pi}{4}\left(\frac{20}{\sqrt[3]{\pi}}\right)^2 + \left(\frac{4000}{\left(\frac{20}{\sqrt[3]{\pi}}\right)}\right)$ = **439 cm²** (3 s.f.)

Practice Questions

Q1 1 litre of water is poured into a bowl. The volume (v) of water in the bowl (in ml) is modelled by the function: $v = 17t^2 + 10t$. Find the rate at which water is poured into the bowl when $t = 4$ seconds.

Q2 The height (h m) a firework can reach is related to the mass (m g) of fuel it carries by: $h = \frac{m^2}{10} - \frac{m^3}{800}$

Find the mass of fuel required to achieve the maximum height and state what the maximum height is.

Exam Questions

Q1 A steam train travels between Haverthwaite and Eskdale at a speed of x miles per hour and burns y units of coal, where y is modelled by: $2\sqrt{x} + \frac{27}{x}$, for $x > 2$.

a) Find the speed that gives the minimum coal consumption. [4 marks]

b) Find $\frac{d^2y}{dx^2}$, and hence show that this speed gives the minimum coal consumption. [2 marks]

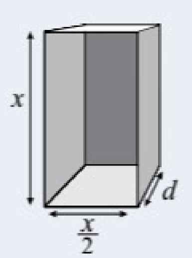
c) Calculate the minimum coal consumption. [1 mark]

Q2 Ayesha is building a closed-back bookcase. She uses a total of 72 m² of wood (not including shelving) to make a bookcase that is x metres high, $\frac{x}{2}$ metres wide and d metres deep, as shown.

a) Show that the full capacity of the bookcase is given by: $V = 12x - \frac{x^3}{12}$. [4 marks]

b) Find the value of x for which V is stationary. Leave your answer in surd form. [3 marks]

c) Show that this is a maximum point and hence calculate the maximum V. [4 marks]



All this page has done is maximise my hunger for pie...

I hope I've managed to convince you that differentiation can pop up pretty much anywhere — those cheeky examiners can make a whole question about it without so much as a single 'd'. Don't fall for their evil schemes.