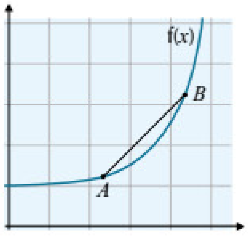
Differentiation

You can also Differentiate from First Principles

You can use this **formula** to find the derivative of a function from **first principles**. I know it looks nasty, but don't worry — it doesn't bite.

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

You might see 'δx' instead of h in the formula, but it means the same thing



- To see where this comes from, imagine the graph of f(x), and a line joining two points on the graph, A and B.
- As B moves closer to A, the gradient of the line AB gets closer to the gradient of the function at A.
- The formula is basically doing the same thing, but instead of the points A and B, you're looking at $(x, f(x))$ and $(x + h, f(x + h))$ — as h gets closer to 0, $x + h$ gets closer to x.

Here's one of our classic step-by-step guides on how to tackle a question like this:

1) Find $\frac{f(x+h)-f(x)}{h}$ and **simplify** (you need to **remove** h from the **denominator** when you're simplifying).
2) Find the **limit** of the expression as h tends to zero (written $\lim_{h \to 0}$) by setting $h = 0$ and simplifying.
3) If needed, put the **x-value** for your given point into the expression to find the **gradient** of f(x) at that point.

Example: Differentiate $f(x) = x^2$ from first principles.

Substitute $f(x) = x^2$ into the formula: $f'(x) = \lim_{h \to 0}\left(\frac{(x+h)^2 - x^2}{h}\right)$

The x^2s cancel on the top, and then you can cancel h from the bottom too.
$= \lim_{h \to 0}\left(\frac{x^2 + 2hx + h^2 - x^2}{h}\right)$
$= \lim_{h \to 0}\left(\frac{2hx + h^2}{h}\right)$

Now you can set $h = 0$ (without dividing by 0) to find the derivative:
$= \lim_{h \to 0}(2x + h)$
$= \mathbf{2x}$ what a shocker...

Practice Questions

Q1 Differentiate these functions with respect to x: a) $y = x^2 + 2$ b) $y = x^4 + \sqrt{x}$ c) $y = \frac{7}{x^2} - \frac{3}{\sqrt{x}} + 12x^3$

Q2 Find the gradient of the graph of $y = x^3 - 7x^2 - 1$ at $x = 2$.

Q3 Find the equations of the tangent and the normal to the curve $y = \sqrt{x^3} - 3x - 10$ at $x = 16$.

Q4 Use differentiation from first principles to find the derivative of $f(x) = 5x$.

Exam Questions

Q1 Find the gradient of the curve $y = \frac{1}{\sqrt{x}} + \frac{1}{x}$ at the point $\left(4, \frac{3}{4}\right)$. [2 marks]

Q2 The curve C is given by the equation $y = mx^3 - x^2 + 8x + 2$, for a constant m.

a) Find $\frac{dy}{dx}$. [1 mark]

The point P lies on C, and has an x-coordinate of 5.
The normal to C at P is parallel to the line given by the equation $y + 4x - 3 = 0$.

b) Find the gradient of curve C at P. [2 marks]

c) Hence or otherwise, find: (i) the value of m, [3 marks]

(ii) the y-coordinate of P. [2 marks]

Q3 Show that the lines $y = \frac{x^3}{3} - 2x^2 - 4x + \frac{86}{3}$ and $y = \sqrt{x}$ both go through the point (4, 2), and are perpendicular at that point. [6 marks]

Q4 Use a binomial expansion to differentiate $f(x) = x^4$ from first principles. [5 marks]

f(x) and g(x) are like identical twins — it can be hard to differentiate them...

This is where A-level maths really kicks off, but don't get carried away and forget the basics. Always write out your working really clearly, particularly when differentiating from first principles. I mean, I know the answer's obvious, and I know you know the answer's obvious, but if they've asked you to use the formula then you'd better do it properly.