Random Events and Venn Diagrams

Random events happen by chance. Probability is a measure of how likely they are. It can be a chancy business.

A Random Event has Various Outcomes

1) In a **trial** (or experiment) the things that can happen are called **outcomes** (so if I time how long it takes to eat my dinner, 63 seconds is a possible outcome).
2) **Events** are 'groups' of one or more outcomes (so an event might be 'it takes me less than a minute to eat my dinner every day one week').
3) When all outcomes are **equally likely**, you can work out the **probability** of an event by **counting** the outcomes:

$$\text{P(event)} = \frac{\text{Number of outcomes where event happens}}{\text{Total number of possible outcomes}}$$

Example: I have a bag with 15 balls in — 5 red, 6 blue and 4 green. I pick a ball without looking. What is the probability the ball is: a) red, b) blue, c) green, d) either red or green?

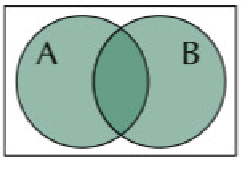
Any ball is **equally likely** to be picked — there are **15 possible outcomes**.
Of these 15 outcomes, 5 are red, 6 are blue and 4 are green.

And so: a) $\text{P(red ball)} = \frac{5}{15} = \frac{1}{3}$ b) $\text{P(blue ball)} = \frac{6}{15} = \frac{2}{5}$ c) $\text{P(green ball)} = \frac{4}{15}$

You can find the probability of **either** red **or** green in a similar way: d) $\text{P(red or green)} = \frac{5+4}{15} = \frac{9}{15} = \frac{3}{5}$

Venn Diagrams show which *Outcomes* correspond to which *Events*

Say you've got 2 events, **A** and **B**. A **Venn diagram** can show which outcomes satisfy event A, which satisfy B, which satisfy both, and which satisfy neither.



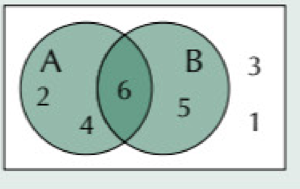
(i) All outcomes satisfying event A go in one part of the diagram, and all outcomes satisfying event B go in another bit.
(ii) If they satisfy '**both A and B**', they go in the dark green middle bit, written **A ∩ B** (and called the **intersection** of A and B).
(iii) The whole of the green area is written **A ∪ B** — it means 'either A or B' (and is called the **union** of A and B).

Again, you can work out probabilities of events by counting outcomes and using the formula above.
You can also get a nice formula linking P(A ∩ B) and P(A ∪ B).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If you just add up the outcomes in A and B, you end up counting A ∩ B twice — that's why you have to subtract it.

Example: If I roll a dice, event A could be 'I get an even number', and B 'I get a number bigger than 4'. The Venn diagram would be:



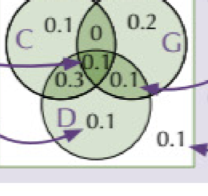
$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{2}{6} = \frac{1}{3}$

$P(A \cap B) = \frac{1}{6}$ $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$

Here, I've just counted outcomes — but I could have used the formula.

Example: A survey was carried out to find what pets people like.
The probability they like dogs is 0.6. The probability they like cats is 0.5.
The probability they like gerbils is 0.4. The probability they like dogs and cats is 0.4.
The probability they like cats and gerbils is 0.1, and the probability they like gerbils and dogs is 0.2.
Finally, the probability they like all three kinds of animal is 0.1.

Draw a Venn diagram to show this information, using C for the event 'likes cats', D for 'likes dogs' and G for 'likes gerbils'.



① Stick in the middle one first — 'likes all 3 animals' (i.e. C ∩ D ∩ G).

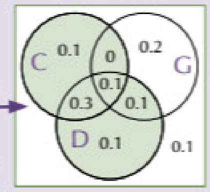
② Then do the 'likes 2 animals' probabilities by taking 0.1 from each given 'likes 2 animals' probability. (If they like 3 animals, they'll also be in the 'likes 2 animals' bits.)

③ Then do the 'likes 1 kind of animal' probabilities, by making sure the total probability in each circle adds up to the probability in the question.

④ Finally, subtract all the probabilities so far from 1 to find 'likes none of these animals'.

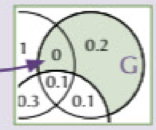
Random Events and Venn Diagrams

Example (cont.): Find: a) the probability that someone likes either dogs or cats,
b) the probability that someone likes gerbils but not dogs,
c) the probability that someone who likes dogs **also** likes cats.

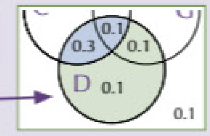


a) From the Venn diagram, the probability that someone likes either dogs or cats is: 0.1 + 0 + 0.1 + 0.3 + 0.1 + 0.1 = **0.7**

b) The probability that someone likes gerbils but not dogs is: 0 + 0.2 = **0.2**



c) For the probability that a dog-lover **also** likes cats, ignore everything outside the 'dogs' circle.



$$\text{P(dog-lover also like cats)} = \frac{0.3 + 0.1}{0.3 + 0.1 + 0.1 + 0.1} = \frac{2}{3}$$

*The **Complement** of 'Event A' is **'Not Event A'***

An event A will either happen or not happen. The event 'A doesn't happen' is called the **complement** of A (or **A′**). On a Venn diagram, it looks like this:

At least one of A and A′ has to happen, so... **P(A) + P(A′) = 1** or **P(A′) = 1 – P(A)**

Example: Nic keeps his socks loose in a box. He picks out a sock. He calculates that the probability of then picking out a matching sock is 0.56. What is the probability of him not picking a matching sock?

Call event A 'picks a matching sock'. Then A′ is 'doesn't pick a matching sock'.
Now A and A′ are **complementary events** (and P(A) = 0.56), so P(A′) = 1 – 0.56 = **0.44**

Example: For two events, A and B: P(A′) = 0.42, P(A ∩ B′) = 0.15, P(B) = 0.55
Use a two-way table to find: a) P(A ∩ B), b) P(A ∪ B′).

	A	A′	Total
B	0.43	0.12	**0.55**
B′	**0.15**	0.3	0.45
Total	0.58	**0.42**	**1**

Start by filling in P(A′) = **0.42**, P(A ∩ B′) = **0.15** and P(B) = **0.55** in the table, then use the fact that the probabilities should add up to **1** to find the rest:

a) P(A ∩ B) is the entry in the A column and the B row — this is **0.43**.

b) P(A ∪ B′) is the sum of all of the entries in either the A column OR the B′ row – so P(A ∪ B′) = 0.43 + 0.15 + 0.3 = **0.88**.
Make sure you don't just add the totals P(A) + P(B′) — you'd be counting the 0.15 twice. I've shaded in the bits of the table you want so that you can see what I mean.

You can also have two-way tables showing numbers of outcomes instead of probabilities. You probably saw these at GCSE.

Practice Questions

Q1 Arabella rolls two standard dice and adds the two results together. What is the probability that she scores:
a) a prime number, b) a square number, c) a number that is either a prime number or a square number?

Q2 Half the students in a sixth-form college eat sausages for dinner and 20% eat chips. 10% of those who eat chips also eat sausages. Show this information in a two-way table, and use it to find the percentage of students who:
a) eat both chips and sausages, b) eat chips but not sausages, c) eat either chips or sausages but not both.

Exam Question

Q1 A soap company asked 120 people about the types of soap (from Brands A, B and C) they bought. Brand A was bought by 40 people, Brand B by 30 people and Brand C by 25. Both Brands A and B were bought by 8 people, B and C were bought by 10 people, and A and C by 7 people. All three brands were bought by 3 people.

a) Represent this information in a Venn diagram. [5 marks]

b) (i) If a person is selected at random, find the probability that they buy at least one of the soaps. [2 marks]
(ii) If a person is selected at random, find the probability that they buy at least two of the soaps. [2 marks]

I took some scales with me to the furniture shop — I like to weigh tables...

I must admit — I kind of like these pages. This stuff isn't too hard, and it's really useful for answering loads of questions. And one other good thing is that Venn diagrams look, well, nice somehow. But more importantly, the thing to remember when you're filling one in is that you usually need to 'start from the inside and work out'.

Mutually Exclusive and Independent Events

Hang on, there was something else to go on this page as well. What was it again... Oh of course — tree diagrams. They blossom from tiny question-acorns into beautiful trees of possibility. How inspiring — not to mention useful...

Tree Diagrams *Show Probabilities for* ***Two or More*** *Events*

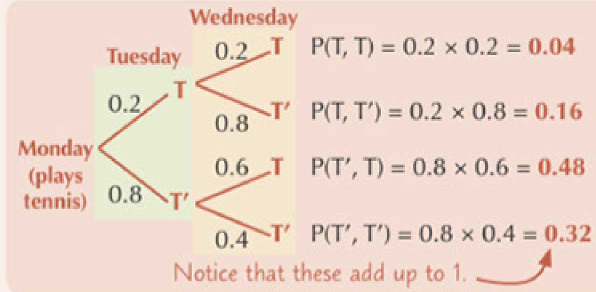
Each 'chunk' of a tree diagram is a **trial**, and each branch of that chunk is a possible **outcome**. Multiplying probabilities along the branches gives you the probability of a **series** of outcomes.

Example: If Susan plays tennis one day, the probability that she'll play the next day is 0.2. If she doesn't play tennis, the probability that she'll play the next day is 0.6. She plays tennis on Monday. What is the probability she plays tennis on the Wednesday of the same week?

Let T mean 'plays tennis', so T′ means 'doesn't play tennis'.
You're interested in **either** P(T, T) **or** P(T′, T).

$$P(\text{plays on Wednesday}) = P(T, T) + P(T', T) = 0.04 + 0.48 = \mathbf{0.52}$$

To find the probability of one event **or** another happening, you have to **add** the probabilities.



Susan was ready for some unlikely outcomes on the tennis court.

Example: A box of biscuits contains 5 chocolate biscuits and 1 lemon biscuit. George takes out 2 biscuits at random, one at a time, and eats them. Find the probability that the second biscuit is chocolate.

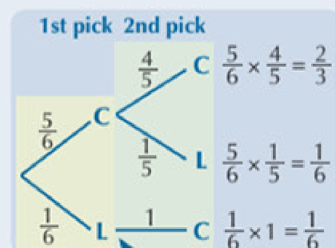
Let C mean 'picks a chocolate biscuit' and let L mean 'picks the lemon biscuit'.
The second biscuit being chocolate is shown by 2 'paths' along the branches — so you can add up the probabilities:

$$P(\text{second biscuit is chocolate}) = \left(\frac{5}{6} \times \frac{4}{5}\right) + \left(\frac{1}{6} \times 1\right) = \frac{2}{3} + \frac{1}{6} = \mathbf{\frac{5}{6}}$$

There's a quicker way to do this, as there's only one outcome where the chocolate **isn't** picked last:

$P(\text{second biscuit is } \textbf{not} \text{ chocolate}) = \frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$, so $P(\text{second biscuit is chocolate}) = 1 - \frac{1}{6} = \mathbf{\frac{5}{6}}$

It's sometimes easier to find the probability of the **complement** of the event you're interested in.



Here there are no lemon biscuits left, so the tree diagram doesn't branch.

If an object is chosen **with replacement**, the probability of choosing a particular item is **the same** for each pick. In the example above, if George puts his first biscuit back instead of eating it, the probability of picking 2 chocolate biscuits becomes:

$$P(C, C) = \frac{5}{6} \times \frac{5}{6} = \mathbf{\frac{25}{36}} > \mathbf{\frac{2}{3}}$$

P(C, C) is slightly greater with replacement. This makes sense — there are more chocolate biscuits available for his 2nd pick, so he is more likely to choose one.

Mutually Exclusive *Events have* ***No Overlap***

If two events **can't both happen** at the same time (i.e. P(A and B) = 0) they're called **mutually exclusive** (or just '**exclusive**'). If A and B are exclusive, then the probability of A **or** B is: P(A or B) = P(A) + P(B).

This is the formula from p.78, with P(A and B) = 0.

More generally,

For n **exclusive** events (i.e. only one of them can happen at a time):
$$P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

Example: Find the probability that a card pulled at random from a standard pack of cards (no jokers) is either a picture card (a Jack, Queen or King) or the 7, 8 or 9 of clubs.

Call **event A** — 'I get a picture card', and **event B** — 'I get the 7, 8 or 9 of clubs'. Then $P(A) = \frac{12}{52}$ and $P(B) = \frac{3}{52}$.
Events A and B are **mutually exclusive** — they can't both happen.
So the probability of either A or B is: $P(A \text{ or } B) = P(A) + P(B) = \frac{12}{52} + \frac{3}{52} = \mathbf{\frac{15}{52}}$

Mutually Exclusive and Independent Events

Independent Events have No Effect on each other

If the probability of B happening doesn't depend on whether or not A has happened, then A and B are **independent**.

This means that: For independent events: **P(A and B) = P(A)P(B)**

Example: V and W are independent events, where P(V) = 0.2 and P(W) = 0.6.
Find: a) P(V and W), b) P(V or W).

a) Just put the numbers into the formula for independent events: P(V and W) = P(V)P(W) = 0.2 × 0.6 = **0.12**

b) Using the formula on page 78: P(V or W) = P(V) + P(W) – P(V and W) = 0.2 + 0.6 – 0.12 = **0.68**

Sometimes you'll be asked if two events are independent or not. Here's how you work it out...

Example: You are exposed to two infectious diseases — one after the other. The probability you catch the first (A) is 0.25, the probability you catch the second (B) is 0.5, and the probability you catch both of them is 0.2. Are catching the two diseases independent events?

Compare P(A and B) and P(A)P(B) — if they're different, the events **aren't independent**.

P(A and B) = 0.2 P(A)P(B) = 0.25 × 0.5 = 0.125

P(A and B) and P(A)P(B) are different, so they're **not independent**.

Practice Questions

Q1 Zofia has 20 cards numbered 1-20. She picks two cards at random, one at a time, without replacement.

a) Are the events 'both numbers are prime numbers' and 'the sum of the numbers is less than 10' mutually exclusive? Explain your answer.

b) Are the events 'the first number is even' and 'the second number is odd' independent? Explain your answer.

Q2 In a school orchestra (made up of pupils in either the upper or lower school), 40% of the musicians are boys. Of the boys, 30% are in the upper school. Of the girls in the orchestra, 50% are in the upper school.

a) Represent this information on a tree diagram.

b) Find the probability that a musician chosen at random is in the upper school.

Q3 For lunch, I eat either chicken or beef for my main course, and either chocolate cake or ice cream for dessert. The probability that I eat chicken is $\frac{1}{3}$, and if I do, the probability that I eat ice cream is $\frac{2}{5}$. If I have beef instead, then the probability that I have ice cream is $\frac{3}{4}$. Find the probability that:

a) I have either chicken or ice cream, but not both, b) I eat ice cream.

Exam Questions

Q1 Event J and Event K are independent events, where P(J) = 0.7 and P(K) = 0.1.

a) Find: (i) P(J and K) [1 mark]
(ii) P(J or K) [2 marks]

b) If L is the event that neither J or K occurs, find P(L and K′). [2 marks]

Q2 A jar contains 3 red counters and 6 green counters. Three random counters are removed from the jar one at a time. The counters are not replaced after they are drawn.

a) Draw a tree diagram to show the probabilities of the various outcomes. [3 marks]

b) Find the probability that the third counter is green. [2 marks]

c) Find the probability that all of the counters are the same colour. [2 marks]

d) Find the probability that at least one counter is red. [2 marks]

All the events that are independent — throw your hands up at me...

Probability questions can be tough. For tricky questions, try drawing a Venn diagram or a tree diagram, even if the question doesn't tell you to — they're really useful for understanding what on earth is going on in a question. And don't forget the definitions of mutually exclusive and independent events — they're key terms you need to know.