

# Constant Acceleration Equations

Welcome to the technical world of kinematics. Fashions may change, but there will always be questions that involve objects travelling in a straight line. It's just a case of picking the right equations to solve the problem.

## There are Five Constant Acceleration Equations

These are also called “**suvat**” questions because of the five variables involved:

$s$  = displacement in m (metres)  
 $u$  = initial velocity in  $\text{ms}^{-1}$   
 $v$  = final velocity in  $\text{ms}^{-1}$   
 $a$  = acceleration in  $\text{ms}^{-2}$   
 $t$  = time taken in s (seconds)

Displacement, velocity and acceleration are vector quantities (see p.136).

**Constant acceleration equations:**

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

You need to know how to derive these using graphs (p.188) and calculus (p.190).

The acceleration must be constant.

You might have to convert units so that they all match.

These equations are on the formula sheet, but you still need to be comfortable using them. Questions might give you **three variables** — your job is to **choose** the equation that will find you an unknown **fourth variable**. Constant acceleration questions often have the following **modelling assumptions** (see p.218 for more on modelling):

- 1) **The object is a particle** — its dimensions can be ignored and no other forces (e.g. air resistance) act on it.
- 2) **Acceleration is constant** — so you can use the suvat equations.

**Example:** A jet ski travels in a straight line along a river. It passes under two bridges 200 m apart and is observed to be travelling at  $5 \text{ ms}^{-1}$  under the first bridge and at  $9 \text{ ms}^{-1}$  under the second bridge. Calculate its acceleration (assuming it is constant).

List the variables:

$s = 200$

$u = 5$

$v = 9$

$a = a$

$t = t$

You have to work out  $a$ .

You're not told about the time taken.

Choose the equation with  $s$ ,  $u$ ,  $v$  and  $a$  in it:

$v^2 = u^2 + 2as$

Substitute values:

$9^2 = 5^2 + (2 \times a \times 200)$

Simplify:

$81 = 25 + 400a$

Rearrange:

$400a = 81 - 25 = 56$

Then solve:

$a = \frac{56}{400} = 0.14 \text{ ms}^{-2}$

Here the units all match — m,  $\text{ms}^{-1}$  and  $\text{ms}^{-2}$ .

## Motion under Gravity just means taking $a = g$

Don't be put off by questions involving objects **moving freely under gravity** — they're just telling you the **acceleration is  $g$** .

If the question gives a mass or weight, then it's a **forces** question — see Section 17.

Assume that  $g = 9.8 \text{ ms}^{-2}$  unless the exam paper says otherwise. If you don't, you risk losing a mark for accuracy.

**Example:** A pebble is dropped into a hole 18 m deep and moves freely under gravity until it hits the bottom. Calculate the time it takes to reach the bottom. (Take  $g = 9.8 \text{ ms}^{-2}$ .)

List the variables:

$s = 18$

$u = 0$

$v = v$

$a = 9.8$

$t = t$

Because the pebble was dropped, not thrown.

$a = g = 9.8 \text{ ms}^{-2}$ , because it's falling freely.

Choose the equation with  $s$ ,  $u$ ,  $a$  and  $t$  in it:  $s = ut + \frac{1}{2}at^2$

Substitute values:  $18 = (0 \times t) + \left(\frac{1}{2} \times 9.8 \times t^2\right)$

Simplify:  $18 = 4.9t^2$

Rearrange to give  $t^2$ :  $t^2 = \frac{18}{4.9} = 3.67\dots$

Take the square root:  $t = \sqrt{3.67\dots} = 1.92 \text{ s (3 s.f.)}$

Time is always positive, so ignore the negative square root.

When an object is projected vertically **upwards**, it helps to choose **up** as the **positive direction**. Gravity always acts **downwards**, so you'd need to use  $a = -g$ .

At the object's **maximum height**,  $v = 0$  — its vertical velocity is momentarily zero before it starts falling again.

# Constant Acceleration Equations

## Sometimes there's More Than One Object Moving at the Same Time

For these questions,  $t$  is often the same (or connected as in this example) because time ticks along at the same rate for both objects. The distance travelled might also be connected.

**Example:** Car A travels along a straight road at a constant velocity of  $30 \text{ ms}^{-1}$ , passing point R at time  $t = 0$ . Exactly 2 seconds later, a second car, B, travelling at  $25 \text{ ms}^{-1}$ , moves in the same direction from point R. Car B accelerates at a constant  $2 \text{ ms}^{-2}$ . Show that, when the two cars are level,  $t^2 - 9t - 46 = 0$ .

For each car, there are different "suvat" variables, so write separate lists and separate equations.

<p style="text-align: center;"><b>CAR A</b></p> <div style="display: flex; justify-content: space-between;"> <div style="border: 1px dashed black; padding: 5px; width: 40%;">             Constant speed so <math>a_A = 0</math> </div> <div style="text-align: center;"> <math>u_A = 30</math>    <math>v_A = 30</math>  <math>a_A = 0</math>    <math>s_A = s</math>  <math>t_A = t</math> </div> </div> <p style="text-align: center;">Distance travelled from point R.</p> <p>The two cars are level, so choose an equation with <math>s</math> in it:</p> $s = ut + \frac{1}{2}at^2$ <p>Substitute values: <math>s = 30t + (\frac{1}{2} \times 0 \times t^2)</math></p> <p>Simplify: <math>s = 30t</math></p>	<p style="text-align: center;"><b>CAR B</b></p> <div style="display: flex; justify-content: space-between;"> <div style="border: 1px dashed black; padding: 5px; width: 40%;"> <math>u_B = 25</math>    <math>v_B = v</math>  <math>a_B = 2</math>    <math>s_B = s</math>  <math>t_B = (t - 2)</math> </div> <div style="text-align: center;"> <math>s</math> is the same for both cars because they're level.         </div> </div> <p style="text-align: center;">B passes R 2 seconds after A.</p> <p>Use the same equation: <math>s = ut + \frac{1}{2}at^2</math></p> <p>Substitute values: <math>s = 25(t - 2) + (\frac{1}{2} \times 2 \times (t - 2)^2)</math></p> <p>Simplify: <math>s = 25t - 50 + (t - 2)(t - 2)</math></p> $s = 25t - 50 + (t^2 - 4t + 4)$ $s = t^2 + 21t - 46$
<p>The distance travelled by both cars is equal, so put the equations for <math>s</math> equal to each other and rearrange: <math>30t = t^2 + 21t - 46</math></p> <p style="text-align: right;"><math>t^2 - 9t - 46 = 0</math>, as required</p>	

## Practice Questions

Take  $g = 9.8 \text{ ms}^{-2}$  in each of these questions.

- Q1 A motorcyclist accelerates uniformly from  $3 \text{ ms}^{-1}$  to  $9 \text{ ms}^{-1}$  in 2 seconds. What is the distance travelled by the motorcyclist during this acceleration?
- Q2 A stone is projected vertically upwards at  $7 \text{ ms}^{-1}$  from the ground. How long does it take to reach its maximum height?
- Q3 A ball is projected vertically upwards at  $3 \text{ ms}^{-1}$  from a point 1.5 m above the ground. How fast will the ball be travelling when it hits the ground?



"Model me as a falling particle!!!!leeee..."

## Exam Questions

- Q1 A window cleaner of a block of flats accidentally drops his sandwich, which falls freely to the ground. The speed of the sandwich as it passes a high floor is  $u \text{ ms}^{-1}$ . After a further 1.2 seconds the sandwich is moving at a speed of  $17 \text{ ms}^{-1}$  past a lower floor.
- a) Find the value of  $u$ . [3 marks]
- b) The vertical distance between the consecutive floors of the building is  $h$  m. It takes the sandwich another 2.1 seconds to fall the remaining 14 floors to the ground. Find  $h$ . [4 marks]
- Q2 A rocket is projected vertically upwards from a point 8 m above the ground at a speed of  $u \text{ ms}^{-1}$  and travels freely under gravity. The rocket hits the ground at  $20 \text{ ms}^{-1}$ . Find:
- a) the value of  $u$ , [3 marks]
- b) how long it takes to hit the ground. [3 marks]

## Newton was one of the first people to realise that gravity sucks...

Make sure you: 1) make a list of the suvat variables EVERY time you get one of these questions, 2) look out for "hidden" values — e.g. "initially at rest..." means  $u = 0$ , 3) choose and solve the equation that goes with the variables you've got.