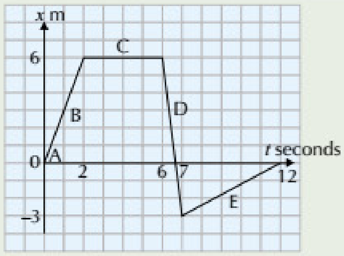
Motion Graphs

You can use displacement-time (x/t), and velocity-time (v/t) graphs to represent all sorts of motion.

Displacement-time Graphs: Height = Distance and Gradient = Velocity

The **steeper** the line, the **greater** the velocity. A **horizontal** line has a **zero gradient**, so the object **isn't moving**.

Example: A rabbit's journey is shown on this x/t graph. Describe the motion.



A: Starts from rest (when $t = 0$, $x = 0$).

B: Travels 6 m in 2 seconds at a velocity of $6 \div 2 = 3$ ms^{-1}.

C: Rests for 4 seconds ($v = 0$).

D: Runs 9 m in 1 second at a velocity of $-9 \div 1 = -9$ ms^{-1} in the opposite direction, passing the starting point.

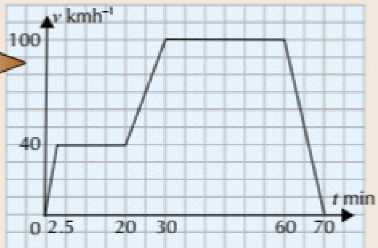
E: Returns to start, travelling 3 m in 5 seconds at a velocity of $3 \div 5 = 0.6$ ms^{-1}.

Velocity is a vector quantity, so the direction needs to be included.

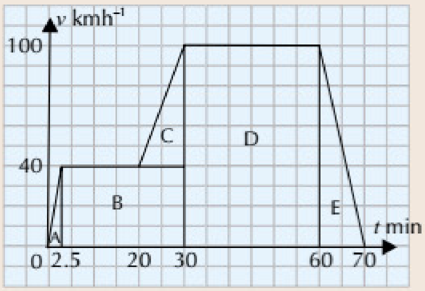
Velocity-time Graphs: Area = Distance and Gradient = Acceleration

The **area** under the graph can be calculated by **splitting** the area into rectangles, triangles or trapeziums. Work out the areas **separately**, then **add** them all up at the end.

Example: A train journey is shown on the v/t graph on the right. Find the distance travelled and the rate of deceleration as the train comes to a stop.



The time is given in minutes and the velocity as kilometres per hour, so divide the time in minutes by 60 to get the time in hours.



Area of A: $(2.5 \div 60 \times 40) \div 2 = 0.833...$

Area of B: $27.5 \div 60 \times 40 = 18.33...$

Area of C: $(10 \div 60 \times 60) \div 2 = 5$

Area of D: $30 \div 60 \times 100 = 50$

Area of E: $(10 \div 60 \times 100) \div 2 = 8.33...$

Total area = 82.5 so distance is **82.5 km**

You might get a speed-time graph instead of a velocity-time graph — they're pretty much the same, except speeds are always positive, whereas velocities can be negative.

Gradient at the end of the journey: -100 kmh^{-1} $\div (10 \div 60)$ hours $= -600$ kmh^{-2}. So the train decelerates at **600 kmh^{-2}**.

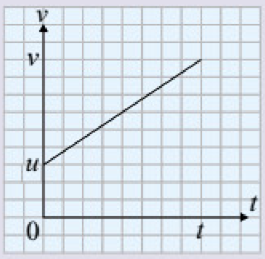
Derive the suvat equations with a Velocity-time Graph

You met the **suvat equations** on p.186 — now it's time to see where they come from. Using a **velocity-time graph** you can derive the equations $v = u + at$ and $s = \frac{1}{2}(u + v)t$, then **use** these to derive the other equations.

Example: The graph shows a particle accelerating uniformly from initial velocity u to final velocity v in t s.

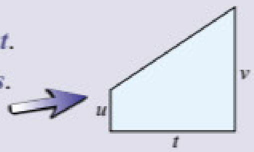
a) Use the graph to derive the equations: (i) $v = u + at$ (ii) $s = \frac{1}{2}(u + v)t$

b) Hence, show that $s = ut + \frac{1}{2}at^2$.



a) (i) It's a v/t graph, so the **gradient** represents the **acceleration**, a. The graph is a **straight line**, crossing the y-axis at u, so using '$y = mx + c$', the equation of the line is $v = u + at$.

(ii) The **area** under a v/t graph represents the **displacement**, s. Here the area is a trapezium, so just use the formula for area of a trapezium: $s = \frac{1}{2}(u + v)t$



b) Substitute $v = u + at$ into $s = \frac{1}{2}(u + v)t$:

$$s = \frac{1}{2}(u + u + at)t = \frac{1}{2}(2u + at)t \Rightarrow s = ut + \frac{1}{2}at^2$$

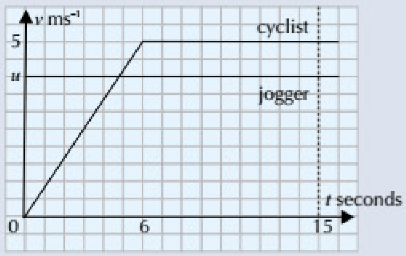
You can derive the other suvat equations in a similar way, or by using calculus — see p.190.

Motion Graphs

Graphs can be used to *Solve Complicated Problems*

Some more complicated problems might involve working out information **not shown directly on the graph**.

Example: A jogger and a cyclist set off at the same time. The jogger runs with a constant velocity. The cyclist accelerates from rest, reaching a velocity of 5 ms^{-1} after 6 s, and then continues at this velocity. The cyclist overtakes the jogger after 15 s. Use the graph below to find the velocity, u, of the jogger.



After 15 s, the distance each has travelled is the same, so you can work out the area under the two graphs to get the distances:

Jogger: distance = area = $15u$

Cyclist: distance = area = $\left(\frac{1}{2} \times 6 \times 5\right) + (9 \times 5) = 60$

So $15u = 60 \Rightarrow u = 4 \text{ ms}^{-1}$

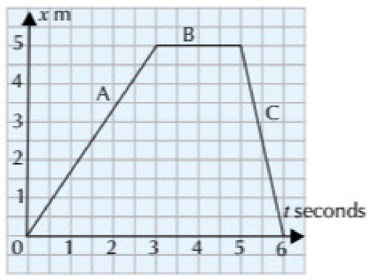
Practice Questions

Q1 Part of an athlete's training drill is shown on the x/t graph to the right.

a) Describe the athlete's motion during the drill.

b) State the velocity of the athlete at $t = 4$.

c) Find the distance travelled by the athlete during the drill.



Q2 A runner starts from rest and accelerates at 0.5 ms^{-2} for 5 seconds. She maintains a constant velocity for 20 seconds then decelerates to a stop at 0.25 ms^{-2}. Find the total distance the runner travelled.

Q3 Using $v = u + at$, and $s = \frac{1}{2}(u + v)t$, show that:

a) $s = vt - \frac{1}{2}at^2$

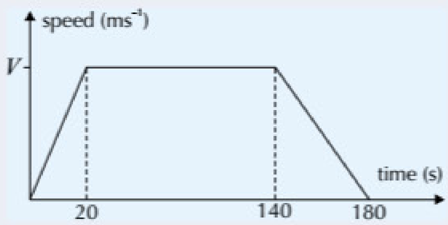
b) $v^2 = u^2 + 2as$

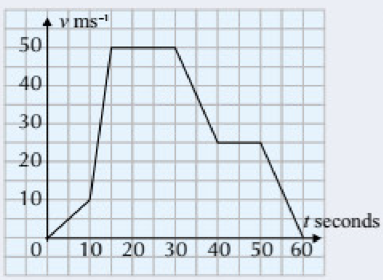
Exam Questions

Q1 A train journey from station A to station B is shown on the graph on the right. The total distance between stations A and B is 2.1 km.

a) Find the value of V. [3 marks]

b) Calculate the distance travelled by the train while decelerating. [2 marks]





Q2 The velocity-time graph of a moving carriage on a roller coaster ride is shown on the left, where v ms^{-1} is the velocity of the carriage.

a) Calculate the acceleration of the carriage at $t = 12$ s. [2 marks]

b) Sean says that the carriage travels further in the first 30 seconds of its journey than the second 30 seconds. Is Sean's statement correct? Provide evidence to support your answer. [4 marks]

c) After T seconds, the carriage has travelled 700 m. Find the value of T. [3 marks]

Random tongue-twister #1 — I wish to wash my Irish wristwatch...

If a picture is worth a thousand words then a graph is worth... um... a thousand and one. Make sure you know the features of each type of graph and know what the gradient and the area under the graph tells you.