Newton's Laws

*That clever chap Isaac Newton established 3 laws involving motion. You need to know **all** of them.*

Newton's Laws of Motion

Newton's First Law	Newton's Second Law	Newton's Third Law
A body will **stay at rest** or **maintain a constant velocity** — unless an extra force acts to **change** that motion.	$F_{net} = ma$ F_{net} (the **overall resultant force**) is equal to the mass multiplied by the acceleration. F_{net} and a act in the same direction.	For **two bodies** in contact with each other, the force each applies to the other is **equal in magnitude** but **opposite in direction**.

$F_{net} = ma$ is sometimes just written as $F = ma$, but it means the same thing.

*Calculating the **Resultant Force** is an **Essential Skill***

Using the first and second laws almost always involves finding the **resultant force**, usually called $\boldsymbol{F_{net}}$. If the resultant force on an object is **zero** then the object is **in equilibrium** (or **at rest**). Calculating the resultant force on an object is just like finding the **resultant vector**, which you saw on p.62-63.

Example: An object is held in equilibrium by the forces (3**i** – 2**j**) N, (–4**i** – 4**j**) N and $\boldsymbol{F}$. Find the magnitude of the force $\boldsymbol{F}$ to 3 significant figures.

The object is in equilibrium, so the resultant force is zero — so both the **i** and **j** components sum to 0:

$3 - 4 + F_i = 0 \Rightarrow F_i = 1$ $\quad -2 - 4 + F_j = 0 \Rightarrow F_j = 6$ $\quad$ So $\boldsymbol{F} = \mathbf{i} + 6\mathbf{j}$ N

Now find the magnitude (see p.64): $|\boldsymbol{F}| = \sqrt{1^2 + 6^2} = \sqrt{37} =$ **6.08 N** (3 s.f.)

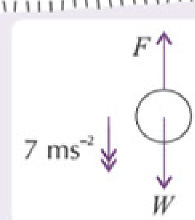
*Weight is given by **Mass × Acceleration Due To Gravity***

A common use of the formula $F = ma$ is calculating the **weight** of an object. An object's weight is a **force** caused by **gravity**. Gravity causes a **constant** acceleration of approximately 9.8 ms^{-2}, denoted $\boldsymbol{g}$.
Putting this into $F = ma$ gives the equation for weight (W): $W = mg$

Remember that weight is a **force** (measured in **newtons**) while mass is measured in **kg** — you might have to **convert units** before using the formula.

g can actually vary a little from 9.8 depending on where you are, but you can always assume it's constant.

Example: A particle of mass 12 kg is acted on by a constant upwards force F. The particle is accelerating vertically downwards at a rate of 7 ms^{-2}.
Find: a) W, the weight of the particle, b) the magnitude of the force F.



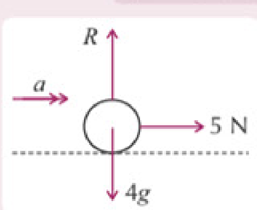
a) Using the formula for weight:
$W = mg$
$= 12 \times 9.8 =$ **117.6 N**

b) Resolving vertically (↓):
$F_{net} = ma \Rightarrow 117.6 - F = 12 \times 7$
$F = 117.6 - 84 =$ **33.6 N**

Resolve Forces in Perpendicular Directions

You might also have to use the constant acceleration equations — you know, the ones from page 96.

Example: A horizontal force of 5 N acts on a mass of 4 kg travelling along a smooth horizontal plane.
a) Find the acceleration of the mass and the normal reaction from the plane. Take $g = 9.8$ ms^{-2}.



Resolve horizontally (→):
$F_{net} = ma$
$5 = 4a$
$\boldsymbol{a = 1.25}$ **ms^{-2} in the direction of the horizontal force**

Always write $F_{net} = ma$ first.

Resolve vertically (↑):
$F_{net} = ma$, so
$R - 4g = 4 \times 0$
$R = 4g =$ **39.2 N**

b) Find the velocity of the particle 6 seconds after it moves off from rest.

You know that: $u = 0$, $v = v$, $a = 1.25$, $t = 6$ $\quad$ Using $v = u + at$: $\quad v = 0 + 1.25 \times 6 =$ **7.5 ms^{-1}**

It started at rest. From part a).

Newton's Laws

You can apply F = ma to i and j Vectors too

Example: a) A particle of mass m kg is acted upon by two forces, $(6\mathbf{i} - \mathbf{j})$ N and $(2\mathbf{i} + 4\mathbf{j})$ N, resulting in an acceleration of magnitude 9 ms^{-2}. Find the value of m.

Resultant force, $F_{net} = (6\mathbf{i} - \mathbf{j}) + (2\mathbf{i} + 4\mathbf{j}) = (8\mathbf{i} + 3\mathbf{j})$ N

Magnitude of $F_{net} = |F_{net}| = \sqrt{8^2 + 3^2} = \sqrt{73} = 8.544...$ N

$F_{net} = ma$, so $8.544... = 9m$

hence $m =$ **0.949 kg** (3 s.f.)

b) The force of $(2\mathbf{i} + 4\mathbf{j})$ N is removed. Calculate the magnitude of the new acceleration.

Magnitude $= \sqrt{6^2 + (-1)^2} = \sqrt{37} = 6.082...$ N

$a = \frac{F}{m} = \frac{6.082...}{0.949...} =$ **6.41 ms^{-2}** (3 s.f.)

You can put **i** and **j** or column vectors straight into $F = ma$ (or $\boldsymbol{F} = m\boldsymbol{a}$).
$\boldsymbol{F}$ and $\boldsymbol{a}$ will both be in vector form, but mass is always scalar.

Example: The resultant force on a particle of mass 2 kg is given by the column vector $\begin{pmatrix}14\\-6\end{pmatrix}$ N. Calculate its velocity vector, 4 seconds after it begins moving from rest.

Using $\boldsymbol{F}_{net} = m\boldsymbol{a}$ with vectors: $\begin{pmatrix}14\\-6\end{pmatrix} = 2\boldsymbol{a} \Rightarrow \boldsymbol{a} = \begin{pmatrix}7\\-3\end{pmatrix}\text{ ms}^{-2}$ — Divide each component by 2.

Now, list the variables you know: $\boldsymbol{u} = \begin{pmatrix}0\\0\end{pmatrix}$, $\boldsymbol{v} = \boldsymbol{v}$, $\boldsymbol{a} = \begin{pmatrix}7\\-3\end{pmatrix}$, $t = 4$ — It starts at rest, so each component of **u** is 0.

Use $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$: $\boldsymbol{v} = \begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}7\\-3\end{pmatrix} \times 4 = \begin{pmatrix}\mathbf{28}\\\mathbf{-12}\end{pmatrix}\text{ ms}^{-1}$

Practice Questions

Q1 A horizontal force of 2 N acts on a 1.5 kg particle initially at rest on a smooth horizontal plane. Find the speed of the particle 3 seconds later.

Q2 A particle of mass 6 g is acted on by a constant force $\boldsymbol{F}$. The particle reaches a velocity of $\begin{pmatrix}200\\125\end{pmatrix}\text{ ms}^{-1}$ 5 seconds after beginning to accelerate from rest. Find the magnitude and direction of $\boldsymbol{F}$.

Q3 Two forces act on a particle of mass 8 kg which is initially at rest on a smooth horizontal plane. The two forces are $(24\mathbf{i} + 18\mathbf{j})$ N and $(6\mathbf{i} + 22\mathbf{j})$ N (with **i** and **j** being perpendicular unit vectors in the plane). Find the magnitude and direction of the particle's resulting acceleration and the magnitude of its displacement after 3 seconds.

Exam Questions

Q1 Two forces, $(x\mathbf{i} + y\mathbf{j})$ N and $(5\mathbf{i} + \mathbf{j})$ N, act on a particle P of mass 2.5 kg. The resultant of the two forces is $(8\mathbf{i} - 3\mathbf{j})$ N.

Find: a) the values of x and y, [2 marks]

b) the magnitude and direction of the acceleration of P, [5 marks]

c) the particle's velocity vector, 5 seconds after it accelerates from rest. [2 marks]

Q2 A skydiver with mass 60 kg falls vertically from rest, experiencing a constant air resistance force, R, as they fall. After 7 seconds, they have fallen 200 m.

a) Find the magnitude of the air resistance on the skydiver to 3 significant figures. [6 marks]

b) State any assumptions that you have made in part a). [2 marks]

Interesting Newton fact: Isaac Newton had a dog called Diamond...

Also, did you know that Isaac Newton and Stephen Hawking both held the same position at Cambridge University? Don't ask me how I know such things, just bask in my amazing knowledge of all things trivial.