Connected Particles

Like Laurel goes with Hardy and Ben goes with Jerry, some particles are destined to be together...

Connected Particles act like One Mass

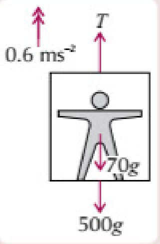
Particles connected together have the **same speeds** and **accelerations** as each other, unless the connection **fails**. If it does, the force connecting them (usually **tension** or **thrust**) will disappear.

Example: A person of mass 70 kg is standing in a lift of mass 500 kg attached to a vertical inextensible, light cable. Given that the lift is accelerating vertically upwards at a rate of 0.6 ms^{-2}, find:
a) T, the tension in the cable, b) the force exerted by the person on the floor of the lift.

You're not given g, so take $g = 9.8$ ms^{-2}.

a) Resolving vertically (↑) for the **whole system**:

$F_{net} = ma$
$T - 570g = 570 \times 0.6$
$T = (570 \times 0.6) + (570 \times 9.8)$
$= \mathbf{5928\ N}$

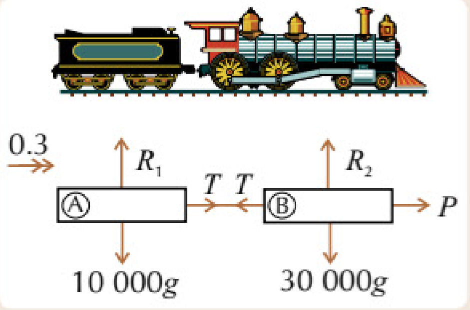


b) Resolving vertically (↑) for the **person** in the lift:

$F_{net} = ma$
$R - 70g = 70 \times 0.6$
$R = 42 + 70g$
$= \mathbf{728\ N}$

You could resolve all the forces on the lift, but it's easier to find the reaction force R on the person from the lift (which has equal magnitude, by Newton's third law).

Example: A 30 tonne locomotive engine is pulling a single 10 tonne carriage as shown. They are accelerating at 0.3 ms^{-2} due to the force P generated by the engine. It's assumed that there are no forces resistant to motion. Find P and the tension in the coupling.



For carriage (A): $F_{net} = ma$
$T = 10\,000 \times 0.3$
$T = \mathbf{3000\ N}$

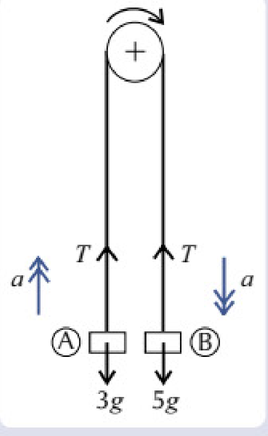
For engine (B): $F_{net} = ma$
$P - T = 30\,000 \times 0.3$
$P - 3000 = 9000$
$P = \mathbf{12\,000\ N}$

You can often resolve forces on each object to get a pair of simultaneous equations that you can solve. They're really easy here though since there's only one force on A.

Pulleys (and 'Pegs') are always Smooth

Out in the real world, things are complicated and scary. In A-level Maths, though, you can always assume that there's no **friction** on a **pulley** or a **peg**, and the **tension** in a **string** will be the **same** either side of it. What a relief.

Example: Masses of 3 kg and 5 kg are connected by an inextensible string and hang vertically either side of a smooth pulley. They are released from rest. Find their acceleration and the time it takes for each to move 40 cm. State any assumptions made in your model. Take $g = 9.8$ ms^{-2}.



For A: $F_{net} = ma$
Resolving upwards: $T - 3g = 3a$ ①

For B: $F_{net} = ma$
Resolving downwards: $5g - T = 5a$
$T = 5g - 5a$ ②

Sub ② into ①: $(5g - 5a) - 3g = 3a$
$a = \mathbf{2.45\ ms^{-2}}$

List variables: $s = 0.4$ m, $u = 0$, $a = 2.45$ ms^{-2}, $t = t$

Use an equation with s, u, a and t in it: $s = ut + \frac{1}{2}at^2$

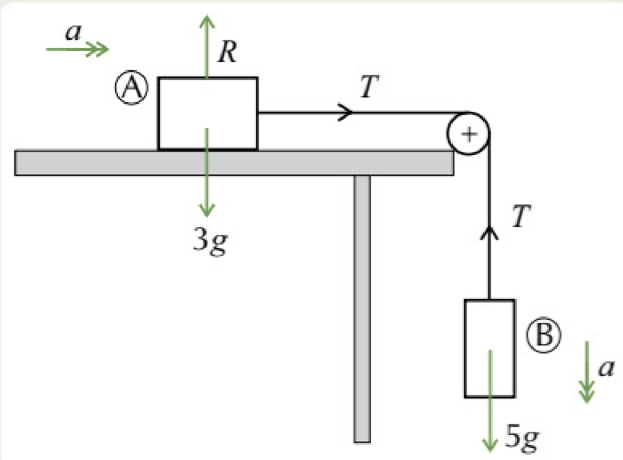
$$0.4 = (0 \times t) + \left(\frac{1}{2} \times 2.45 \times t^2\right) \quad \text{So } t = \sqrt{\frac{0.8}{2.45}} = \mathbf{0.571\ s}\ (3\text{ s.f.})$$

Assumptions: The 3 kg mass does not hit the pulley; there's no air resistance; the string is 'light' so the tension is the same for both A and B, and it doesn't break; the string is inextensible so the acceleration is the same for both masses.

Connected Particles

Use F = ma in the ***Direction Each Particle Moves***

Example: A mass of 3 kg is placed on a smooth horizontal table. A light inextensible string connects it over a smooth peg to a 5 kg mass which hangs vertically as shown. Find the tension in the string if the system is released from rest. Take $g = 9.8\ \text{ms}^{-2}$.



For A:

Resolve horizontally: $F_{net} = ma$

$T = 3a$

$a = \frac{T}{3}$ ①

For B:

Resolve vertically: $F_{net} = ma$

$5g - T = 5a$ ②

Sub ① into ②: $5g - T = 5 \times \frac{T}{3}$

So $\frac{8}{3}T = 5g$

$T = 18.4\ \text{N}$ (3 s.f.)

Practice Questions

Q1 A person of mass 60 kg is standing in a lift of mass 600 kg connected to a light, inextensible vertical wire. The force exerted on them by the lift while it is moving is 640 N. Find the acceleration of the lift and the tension in the wire.

Q2 Two particles are connected by a light inextensible string, and hang in a vertical plane either side of a smooth pulley. When released from rest the particles accelerate at $1.2\ \text{ms}^{-2}$. If the heavier mass is 4 kg, find the weight of the other.

The state of these wires is certainly resulting in a **lot** of tension (don't try this at home).

Exam Questions

Q1 A car of mass 1500 kg is pulling a caravan of mass 500 kg. They experience resistance forces totalling 1000 N and 200 N respectively. The forward force generated by the car's engine is 2500 N. The coupling between the two does not break.

a) Find the acceleration of the car and caravan. [2 marks]

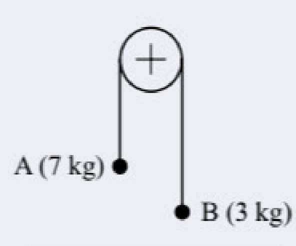
b) Find the tension in the coupling. [2 marks]

Q2 Two particles A and B are connected by a light inextensible string which passes over a smooth fixed pulley as shown. A has a mass of 7 kg and B has a mass of 3 kg. The particles are released from rest with the string taut, and A falls freely until it strikes the ground travelling at a speed of $5.9\ \text{ms}^{-1}$. A does not rebound after hitting the floor.

a) Find the time taken for A to hit the ground. [4 marks]

b) How far will B have travelled when A hits the ground? [2 marks]

c) Find the time (in s) from when A hits the ground until the string becomes taut again. [4 marks]



Connected particles — together forever... isn't it beautiful?

It makes things a lot easier when you know that connected particles act like one mass, and that you won't have to deal with rough pulleys. Those examiners occasionally do try to make your life easier, honestly (if only a little bit).