Cubics

Remember how much you enjoyed factorising quadratics? Well factorising cubics is a little bit similar — only harder, better, faster, stronger. Okay maybe just harder, but at least you still get to do a bit of sketching.

Factorising** a cubic given **One Factor

A **cubic** function has an x^3 term as the highest power. When you factorising a cubic, you put it into (up to) three **brackets**. If the examiners are feeling nice they'll give you **one** of the factors, which makes it a bit **easier** to factorise.

Example: Given that $(x+2)$ is a factor of $f(x) = 2x^3 + x^2 - 8x - 4$, express $f(x)$ as the product of three linear factors.

The first step is to find a quadratic factor. So write down the factor you know, along with another set of brackets:

$(x + 2)(\qquad\qquad) = 2x^3 + x^2 - 8x - 4$

Put the x^2 bit in this new set of brackets. These have to multiply together to give you this:

$(x + 2)(2x^2 \qquad\qquad) = 2x^3 + x^2 - 8x - 4$

Find the number for the second set of brackets. These have to multiply together to give you this:

$(x + 2)(2x^2 \qquad - 2) = 2x^3 + x^2 - 8x - 4$

These multiply to give you $-2x$, but there's $-8x$ in f(x) — so you need an 'extra' $-6x$. And that's what this $-3x$ is for:

$(x + 2)(2x^2 - 3x - 2) = 2x^3 + x^2 - 8x - 4$

You only need $-3x$ because it's going to be multiplied by 2 which makes $-6x$.

Before you go any further, check that there are the same number of x^2's on both sides:

$4x^2$ from here...

$(x + 2)(2x^2 - 3x - 2) = 2x^3 + x^2 - 8x - 4$

...and $-3x^2$ from here add together to give this x^2.

If this is okay, factorise the quadratic into two linear factors.

$2x^2 - 3x - 2 = (2x + 1)(x - 2)$

And so... $2x^3 + x^2 - 8x - 4 = (x + 2)(2x + 1)(x - 2)$

Factorising Cubics

1) Write down the **factor** $(x - k)$.
2) Put in the **x^2 term**.
3) Put in the **constant**.
4) Put in the x term by **comparing** the **number of x's** on both sides.
5) **Check** there are the same **number of x^2's** (or **x's**) on both sides.
6) **Factorise** the quadratic you've found — if that's possible.

If every term in the cubic contains an 'x' (i.e. $ax^3 + bx^2 + cx$) then just take out x as your first factor before factorising the remaining quadratic as usual.

If you wanted to solve a cubic, you'd do it exactly the same way — put it in the form $ax^3 + bx^2 + cx + d = 0$ and factorise.

*Use the **Factor Theorem** to factorise a cubic given **No Factors***

If the nasty examiner has given you **no factors**, you can find one using the Factor Theorem (see p.11) and then use the **method** above to factorise the rest. As if by magic, here's a reminder of the **Factor Theorem**:

If $f(x)$ is a polynomial, and $f(k) = 0$, then $(x - k)$ is a **factor** of $f(x)$.

Example: Factorise $f(x) = 2x^3 + x^2 - 8x - 4$ fully.

Try small numbers until f(something) = 0: For example, calculate f(1), f(−1), f(2), f(−2), etc.

$f(1) = 2(1^3) + 1^2 - 8(1) - 4 = -9$

$f(-1) = 2(-1)^3 + (-1)^2 - 8(-1) - 4 = 3$

$f(2) = 2(2^3) + 2^2 - 8(2) - 4 = 0$

Using the Factor Theorem: $f(2) = 0$, so $(x - 2)$ is a factor.

Write down another set of brackets: $(x - 2)(\qquad\qquad) = 2x^3 + x^2 - 8x - 4$

Use the method described above to get: $2x^3 + x^2 - 8x - 4 = (x - 2)(2x + 1)(x + 2)$

This is actually the same example as above but the working will be slightly different because you're starting with the factor $(x - 2)$.