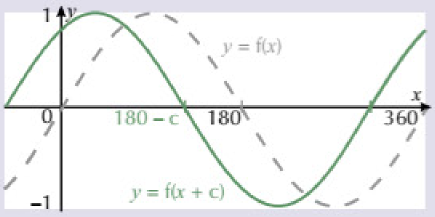
Graphs of Functions

There are *Four* main *Graph Transformations*

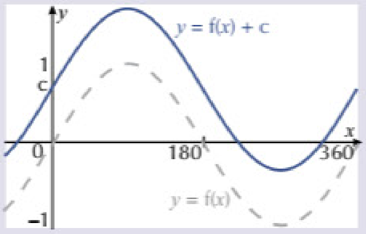
You'll have come across graph transformations before — **translations** (adding things to **shift** the graph vertically or horizontally) and **reflections** in the x- or y- axis. You also need to know **stretches** (either vertical or horizontal). Each transformation has the same effect on any function — here they're applied to $f(x) = \sin x$:

$y = f(x + c)$



For c > 0,
f(x + c) is f(x) **shifted c to the left**,
and f(x – c) is f(x) **shifted c to the right**.

$y = f(x) + c$

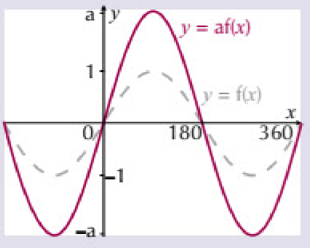


For c > 0, f(x) + c is f(x) **shifted c upwards**, and f(x) – c is f(x) **shifted c downwards**.

Don't forget to shift any asymptotes as well — e.g. the graph of $y = \frac{1}{x + a} + b$ has asymptotes at $y = b$ and $x = -a$.

Reflections in the x-axis flip f(x) vertically and reflections in the y-axis flip f(x) horizontally.

$y = af(x)$

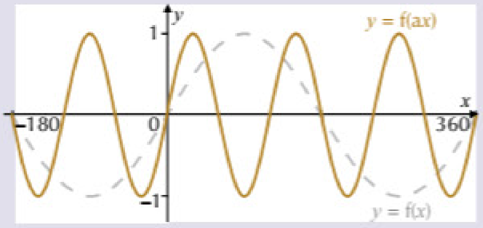


If $|a| > 1$, the graph of $af(x)$ is $f(x)$ **stretched vertically** by a factor of a.

If $0 < |a| < 1$, the graph is **squashed vertically**.

And if $a < 0$, the graph is also **reflected** in the **x-axis**.

$y = f(ax)$



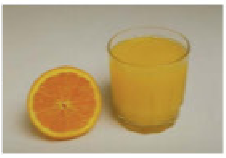
If $|a| > 1$, the graph of $f(ax)$ is $f(x)$ **squashed horizontally** by a factor of a.

If $0 < |a| < 1$, the graph is **stretched horizontally**.

And if $a < 0$, the graph is also **reflected** in the **y-axis**.

A squash by a factor of a is really a stretch by a factor of $\frac{1}{a}$.

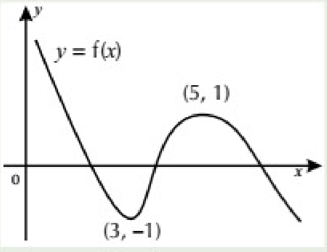
$\frac{1}{2}$f(2 × orange) =



Do *Combinations* of transformations *One at a Time*

Combinations of transformations can look a bit tricky, but if you take them **one step** at a time they're not too bad. Don't do **all** the transformations at once — break it up into **separate bits** (as above) and draw a **graph** for **each stage**.

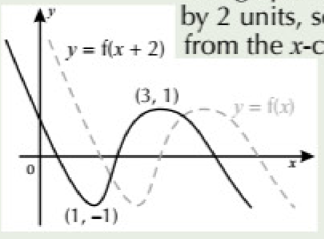
Example: The graph below shows the function $y = f(x)$. Draw the graph of $y = 3f(x + 2)$, showing the coordinates of the turning points.



Make sure you do the transformations the right way round — you should do the bit in the brackets first.

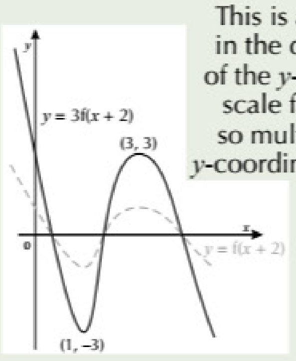
Don't try to do everything at once. First draw the graph of y = f(x + 2) and work out the coordinates of the turning points.

The graph is shifted left by 2 units, so subtract 2 from the x-coordinates.



Now use your graph of y = f(x + 2) to draw the graph of y = 3f(x + 2).

This is a stretch in the direction of the y-axis with scale factor 3, so multiply the y-coordinates by 3.

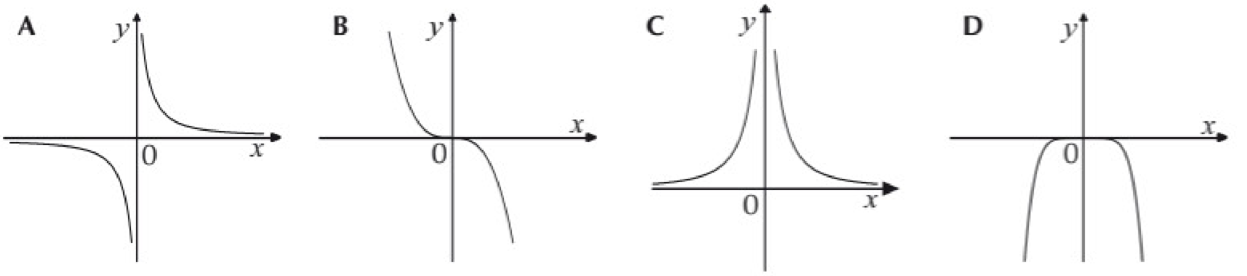


Graphs of Functions

Practice Questions

Q1 Four graphs, A, B, C and D, are shown below. Match each of the following functions to one of the graphs.

a) $y = \frac{4}{x^4}$ b) $y = -3x^6$ c) $y = -1.5x^3$ d) $y = \frac{2}{3x}$

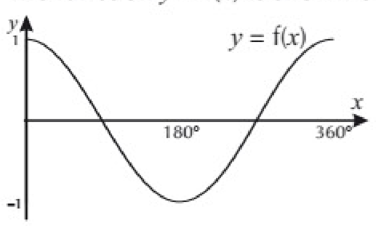


Q2 Sketch the following curves, labelling any points of intersection with the axes:

a) $y = -2x^4$ b) $y = \frac{7}{x^2}$ c) $y = -5x^3$ d) $y = -\frac{2}{x^5}$

Q3 Sketch the graph of $y = f(x)$, where $f(x) = x^2(x + 3)^2$.

Q4 The function $y = f(x)$ is shown on the graph below.



Sketch the graphs of the following:

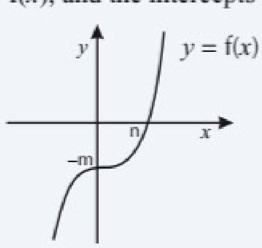
a) $y = \frac{1}{4}f(x)$ b) $y = f(x + 180°)$ c) $y = 2f(x) + 1$

Exam Questions

Q1 $f(x) = (1 - x)(x + 4)^3$

Sketch the graph of $y = f(x)$, labelling the points where the curve intersects the x- and y-axes. [4 marks]

Q2 The graph below shows the curve $y = f(x)$, and the intercepts of the curve with the x- and y-axes.



Sketch the graphs of the following transformations on separate axes, clearly labelling the points of intersection with the x- and y-axes in terms of m and n.

a) $y = f(3x)$ [2 marks]

b) $y = |f(x)|$ [2 marks]

c) $y = -3f(x)$ [2 marks]

d) $y = f(|x|)$ [2 marks]

"Let's get graphical, graphical. I want to get graphical"...

Graphs of $y = kx^n$ and quartics are probably less likely to come up than quadratics or cubics. But if you're struggling to remember the right shape of any graph, test different x-values (e.g. positive values, negative values, values either side of any roots). For graph transformations you might find it useful to remember that stuff outside the brackets affects f(x) vertically and stuff inside affects f(x) horizontally. Now get out there and get sketching (graphs).