You can **Differentiate sin** and **cos** from **First Principles**

You saw how to differentiate a function from **first principles** back on p.86. You can do this for sin and cos too, but you'll need to dust off your **small angle approximations** (p.69) and your **addition formulas** (p.70).

Example: Differentiate $f(x) = \sin x$ from first principles.

Start by writing out the formula: $f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$

Substitute $f(x) = \sin x$: $= \lim_{h \to 0}\left(\frac{\sin(x+h) - \sin x}{h}\right)$

Expand $\sin(x + h)$ with the **addition formula**: $= \lim_{h \to 0}\left(\frac{(\sin x \cos h + \cos x \sin h) - \sin x}{h}\right)$

It's helpful to collect the $\sin x$ and $\cos x$ terms here: $= \lim_{h \to 0}\left(\frac{\sin x(\cos h - 1) + \cos x(\sin h)}{h}\right)$

You're interested in when h gets really small (as $h \to 0$), so you can use the **small angle approximations** ($\sin h \approx h$, $\cos h \approx 1 - \frac{1}{2}h^2$): $= \lim_{h \to 0}\left(\frac{\left(-\frac{1}{2}h^2\right)\sin x + h\cos x}{h}\right)$

h appears on the top and bottom of the fraction, so cancel it: $= \lim_{h \to 0}\left(-\frac{1}{2}h\sin x + \cos x\right)$

As $h \to 0$, $\frac{1}{2}h\sin x \to 0$, so it disappears: $= \cos x$