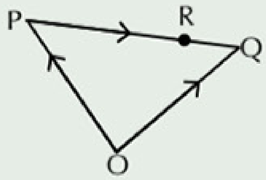
3D Vectors

*Break up **3D Vector** Problems into **Smaller Chunks***

Visualising 3D vector problems can be quite hard. So, if you're given a **difficult** 3D vector problem with **multiple steps**, it helps to break it down into **chunks** and draw them with simple **2D diagrams**.

Example: Points P and Q have position vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ respectively. Point R divides the line PQ in the ratio 3:1. Find the position vector of R.

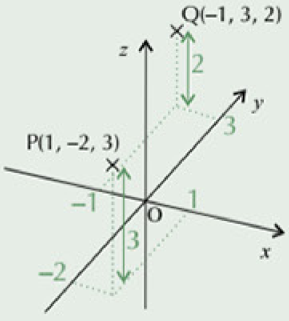
First you need to find the vector $\overrightarrow{PQ}$:



$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (-1 - 1)\mathbf{i} + (3 - (-2))\mathbf{j} + (2 - 3)\mathbf{k}$

$= -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$



R divides $\overrightarrow{PQ}$ in the ratio 3:1, so R is $\frac{3}{4}$ of the way along $\overrightarrow{PQ}$.

This means $\overrightarrow{PR} = \frac{3}{4}\overrightarrow{PQ} = \frac{3}{4}(-2\mathbf{i} + 5\mathbf{j} - \mathbf{k}) = -\frac{3}{2}\mathbf{i} + \frac{15}{4}\mathbf{j} - \frac{3}{4}\mathbf{k}$.

So $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + -\frac{3}{2}\mathbf{i} + \frac{15}{4}\mathbf{j} - \frac{3}{4}\mathbf{k}$

$= \left(1 - \frac{3}{2}\right)\mathbf{i} + \left((-2) + \frac{15}{4}\right)\mathbf{j} + \left(3 - \frac{3}{4}\right)\mathbf{k} = -\frac{1}{2}\mathbf{i} + \frac{7}{4}\mathbf{j} + \frac{9}{4}\mathbf{k}$

You could also do $\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR}$.

Practice Questions

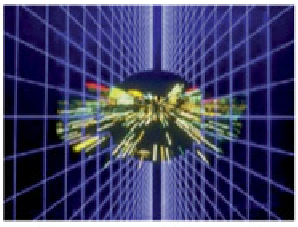
Q1 Point P has the coordinates (7, −3, −2). Find the position vector of P. Give your answer in **unit vector** form.

Q2 Find the **magnitudes** of these vectors: a) $3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ b) $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

Q3 Show that the vectors $\mathbf{v} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ are parallel.

Q4 Find $\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$ where $\mathbf{a} = 3\mathbf{i} + 7\mathbf{j} + \mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

Q5 The position vectors of point S and T are $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ respectively. Calculate the length of the line ST.



If vectors could dream, it would look something like this (probably)...

Exam Questions

Q1 Points P, Q and R have position vectors $\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 7 \\ -5 \end{pmatrix}$ respectively.
Show that $\overrightarrow{OP}$ is parallel to $\overrightarrow{QR}$. [2 marks]

Q2 A skateboard ramp has vertices modelled with position vectors $O = (0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k})$ m, $X = (3\mathbf{i} - \mathbf{j})$ m, $Y = 5\mathbf{j}$ m and $Z = 4\mathbf{k}$ m. Calculate the distance from the midpoint of OZ to the midpoint of XY to 3 s.f. [3 marks]

Q3 a) The points A and B have position vectors $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $-3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ respectively.
Show that $\overrightarrow{AB} = -5\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$. [1 mark]

b) The point M divides the line AB in the ratio 2:1. Calculate the distance of M from the origin. [5 marks]

Vectors let you flit between dimensions like your favourite sci-fi hero...

What do you mean you don't have a favourite sci-fi hero? Urgh, you haven't lived. Three dimensions doesn't really make things much more difficult — it just gives you an extra number to calculate with. You add, subtract and multiply 3D column vectors in the same way as 2D ones — you just have three rows to deal with.