Mutually Exclusive and Independent Events

Modelling with Probabilities Involves *Assumptions*

In most models, the probability of an event is based on some **assumptions**.

For example, when tossing a coin, you assume that each side is **equally likely** to come up, so the probability is 0.5 for each. But it's possible that the coin is **biased**, giving the two outcomes **different probabilities**.

Evaluating and **criticising** the assumptions being made is an important part of the modelling process.

Some common issues to think about are:

- Have you assumed that two (or more) events are **equally likely**? Is this true? Could the probabilities be **biased** in some way?
- Is the probability based on **past data**? Is the data **appropriate**? How **reliable** is the data? How was the data **sampled**?
- Is the experiment itself **truly random**? Is there anything about the way that the experiment is being **carried out** that could affect the outcome?

You'll see on page 170 that choosing a probability distribution also involves making assumptions.

Me, back in my modelling days.

Example: Sanaa wants to know the probability that it will rain tomorrow. She looks up the weather data for the previous 30 days, and finds that it has rained on 12 of them. She concludes that the probability that it will rain is $\frac{12}{30} = 0.4$. Give a reason why this model might be inaccurate.

Sanaa has used the relative frequency of the event to estimate the probability — you might have seen this at GCSE.

There are lots of answers you could give. For example:

- She has only taken data from the past 30 days, which might not be a **large enough sample** to give an accurate estimate, or might not take **seasonal variations** into account.
- She has assumed that the probability that it rains on one day is **not affected** by whether or not it rained the day before (i.e. that they are **independent events**) but this might not be true.

Practice Questions

Q1 Zofia has 20 cards numbered 1-20. She picks two cards at random, one at a time, without replacement.

a) Are the events 'both numbers are prime numbers' and 'the sum of the numbers is less than 10' mutually exclusive? Explain your answer.

b) Are the events 'the first number is even' and 'the second number is odd' independent? Explain your answer.

Q2 Two candidates, A and B, are standing in an election. Two weeks before the vote, a polling company surveys a random sample of people who say they voted in the previous election to ask who they will vote for. Based on the responses, the company predicts that candidate A has a 75% probability of winning. Suggest two assumptions that have been made which may mean this model is inaccurate.

Exam Questions

Q1 Event J and Event K are independent events, where $P(J) = 0.7$ and $P(K) = 0.1$.

a) Find: (i) $P(J \cap K)$ [1 mark]

(ii) $P(J \cup K)$ [2 marks]

b) If L is the event that neither J or K occurs, find $P(L \mid K')$. [4 marks]

Q2 Erwin drives a delivery van for a company that sells fragile glass sculptures of cats. Erwin often has to drive quickly to deliver the sculptures on time. Sometimes, when the customer opens the box, the sculpture is broken. The probability that a sculpture is delivered on time is 0.8. The probability that a sculpture is broken, given that it is delivered on time is 0.56.

a) Erwin claims that the probability of a sculpture being broken is not affected by whether or not he delivers it on time. If Erwin is correct, what is the probability that a sculpture is broken when it is delivered? [2 marks]

b) Erwin is in fact incorrect, and the probability that any sculpture is broken when it is delivered is 0.5. Find the probability that a sculpture was delivered late, given it wasn't broken when it was delivered. [6 marks]

EXCLUSIVE — assumptions made about model at independent event...

Probability questions can be tough. For tricky questions, try drawing a Venn diagram or a tree diagram, even if the question doesn't tell you to — they're really useful for understanding what on earth is going on in a question. And don't forget the definitions of mutually exclusive and independent events — they're key terms you need to know.