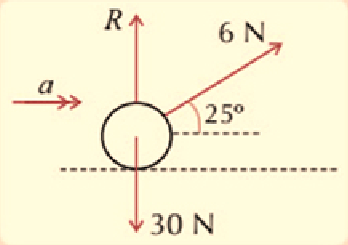
Example: A particle of weight 30 N is being accelerated across a smooth horizontal plane by a force of 6 N acting at an angle of 25° to the horizontal, as shown. Given that the particle starts from rest, find:

a) its speed after 4 seconds b) the magnitude of the normal reaction with the plane.



a) Resolve horizontally (→):

$F_{net} = ma$

$6 \cos 25° = \frac{30}{g}a$

W = mg, so the mass of the particle is $m = \frac{W}{g}$.

$\Rightarrow a = 1.776... \text{ ms}^{-2}$

$v = u + at$

$v = 0 + 1.776... \times 4 =$ **7.11 ms⁻¹** (3 s.f.)

b) Resolve vertically (↑):

There's no vertical acceleration.

$F_{net} = ma$

$R + 6 \sin 25° - 30 = \frac{30}{g} \times 0$

So $R = 30 - 6 \sin 25°$

$=$ **27.5 N** (3 s.f.)

Resolving Forces

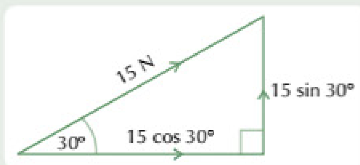
Forces have direction and magnitude, which makes them vectors — this means that you'll need all the stuff you learnt about vectors in Section 10. Oh come on, don't give me that look — it'll be fun, trust me.

*Forces have **Components***

You've done a fair amount of **trigonometry** already, so this should be as straightforward as watching paint dry.

Example: A particle is acted on by a force of 15 N at 30° above the horizontal. Find the horizontal and vertical components of the force in both **i** and **j** form, and as a column vector.

A bit of trigonometry is all that's required:



Force = 15 cos 30°**i** + 15 sin 30°**j**

$= (13.0\mathbf{i} + 7.5\mathbf{j})$ N (3 s.f.) $= \begin{pmatrix} 13.0 \\ 7.5 \end{pmatrix}$ N

(i.e. 13.0 N to the right and 7.5 N upwards)

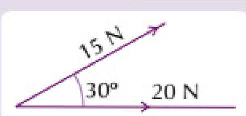
This is also known as **resolving** a force into components.

Add** the **Components** to get the **Resultant

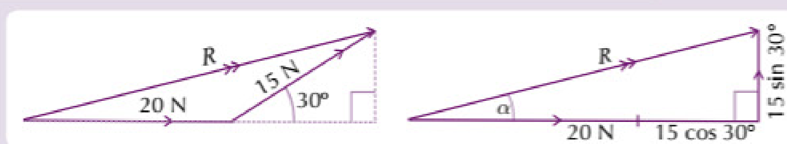
The **resultant force** on an object is the **resultant vector** of all of the forces acting on it. To work it out, you can either use the **'nose to tail'** method you saw back in Section 10, or **resolve** the vectors into horizontal and vertical components and add them together.

See p.136 for more about resultant vectors.

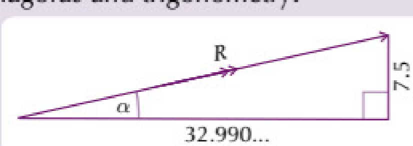
Example: A second horizontal force of 20 N to the right is also applied to the particle in the example above. Find the magnitude and direction of the resultant of these forces.



Put the arrows nose to tail:



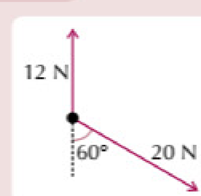
Using Pythagoras and trigonometry:



$R = \sqrt{(32.990...)^2 + 7.5^2}$ = **33.8 N** (3 s.f.)

$\alpha = \tan^{-1}\left(\frac{7.5}{32.99}\right)$ = **12.8°** (1 d.p.) above the horizontal

Example: Find, in **i** and **j** vector form, the resultant force acting on the particle shown below.



Resolve the forces into horizontal (**i**) and vertical (**j**) components:

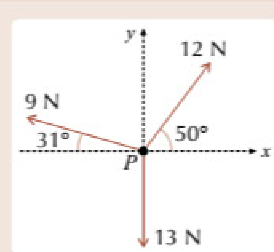
The 12 N force is vertical, so it has components 0**i** + 12**j**

The components of the 20 N force are: 20 sin 60°**i** – 20 cos 60°**j** = 17.320...**i** – 10**j**

Now add the vectors together:

Resultant = (0 + 17.320...)**i** + (12 – 10)**j** = **(17.3i + 2j) N** (3 s.f.)

Example: Three forces of magnitudes 9 N, 12 N and 13 N act on a particle *P* in the directions shown in the diagram. Find the magnitude and direction of the resultant of the three forces, to 2 d.p.



One of the forces is already aligned with the *y*-axis, so it makes sense to start by resolving the other forces relative to this.

Along the *y*-axis: resultant = 9 sin 31° + 12 sin 50° – 13
= 4.635... + 9.192... – 13 = **0.83 N** (2 d.p.)

Along the *x*-axis: resultant = 12 cos 50° – 9 cos 31°
= 7.713... – 7.714... = **0.00 N** (2 d.p.)

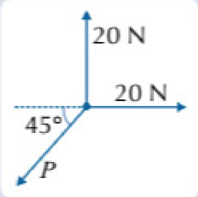
Overall: **0.83 N** (2 d.p.) in the direction of the *y*-axis

Resolving Forces

Particles in *Equilibrium* have *No Resultant Force*

When the resultant force acting on an object is **zero** (in all directions), then the object is **in equilibrium**.
A particle that's in equilibrium either stays still, or moves at a constant speed (see p.203).
More often than not, you'll be told that something's in equilibrium and have to find an **unknown force**.

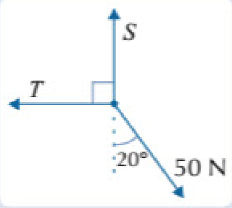
Examples: a) Two perpendicular forces of magnitude 20 N act on a particle. A third force, P, acts at 45° to the horizontal, as shown. Given that the particle is in equilibrium, find the magnitude of P.



Use an arrow to show which direction you're taking as positive.

Resolving horizontally (→): $20 - P\cos 45° = 0 \Rightarrow P = 20 \div 0.707... =$ **28.3 N** (3 s.f.)

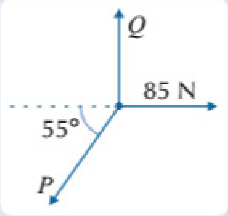
b) A force of 50 N acts on a particle at an angle of 20° to the vertical, as shown. Find the magnitude of the two other forces, T and S, if the particle is in equilibrium.



Resolving vertically (↑): $S - 50\cos 20° = 0 \Rightarrow S =$ **47.0 N** (3 s.f.)

Resolving horizontally (←): $T - 50\sin 20° = 0 \Rightarrow T =$ **17.1 N** (3 s.f.)

c) Three forces act upon a particle. A force of magnitude 85 N acts horizontally, the force Q acts vertically, and the force P acts at 55° to the horizontal, as shown. The particle is in equilibrium. Find the magnitude of P and Q.



Resolving horizontally (←): $P\cos 55° - 85 = 0 \Rightarrow P =$ **148 N** (3 s.f.)

Resolving vertically (↑): $Q - P\sin 55° = 0 \Rightarrow Q =$ **121 N** (3 s.f.)

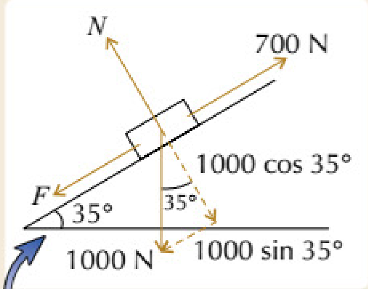
An *Inclined Plane* is a *Sloping Surface*

Now don't be silly — you know that's not what I mean.

Things are a little trickier when you're dealing with **inclined planes**.
In fact, I'm inclined to say that they're far from plain sailing...

Example: A sledge of weight 1000 N is being held on a rough inclined plane at an angle of 35° by a force of 700 N acting parallel to the slope. Find the normal contact force N and the frictional force F acting on the sledge.

The sledge is being held in place — that's your hint that it's in **equilibrium**.



You can use angle properties to show that the two marked angles are always the same.

Apart from the weight (which always acts vertically downwards), all the forces here act either **parallel** or **perpendicular** to the **slope**, so it makes sense to resolve in these directions.

Perpendicular to the slope (↖):

$N - 1000\cos 35° = 0$

$\Rightarrow N = 1000\cos 35° =$ **819 N** (3 s.f.)

Parallel to the slope (↗):

$700 - F - 1000\sin 35° = 0$

$\Rightarrow F = 700 - 1000\sin 35° =$ **126 N** (3 s.f.)

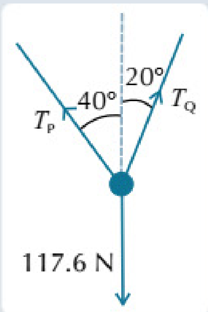
You don't have to resolve forces horizontally and vertically — for inclined planes, it's usually easier to resolve perpendicular and parallel to the plane.

Resolving Forces

Objects on *Strings* produce *Tension*

You could be told the mass of the object instead of the weight — see the next page for more.

Example: A object of weight 117.6 N is held by two light strings, P and Q, acting at 40° and 20° to the vertical, as shown. Find the tension in each string.



Resolving horizontally (←):

$T_P \sin 40° - T_Q \sin 20° = 0 \Rightarrow T_P = T_Q \frac{\sin 20°}{\sin 40°}$ — call this equation (1)

Resolving vertically (↑):

$T_P \cos 40° + T_Q \cos 20° - 117.6 = 0$ — call this equation (2)

Substituting (1) into (2):

$T_Q \frac{\sin 20°}{\sin 40°} \cos 40° + T_Q \cos 20° - 117.6 = 0 \Rightarrow 0.407...T_Q + 0.939...T_Q = 117.6$

$\Rightarrow T_Q = 117.6 \div (1.347...) =$ **87.3 N** (3 s.f.)

Substitute the value of T_Q into (1) to get: $T_P = 87.28... \times 0.532... =$ **46.4 N** (3 s.f.)

Forces can be given in different *Vector Forms*

As well as working with forces given in '**magnitude and direction**' form, be prepared for **i** and **j** or **column** vectors.

Example: Two forces, $A = (2\mathbf{i} - 11\mathbf{j})$ N and $B = (7\mathbf{i} + 5\mathbf{j})$ N, act on a particle. Find the exact magnitude of the resultant force, R, on the particle.

Since the forces are already given in components, finding the resultant force is easy:

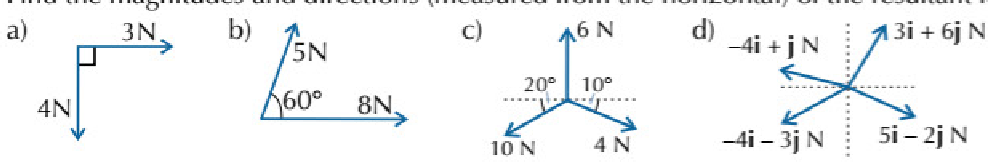
$R = A + B = (2\mathbf{i} - 11\mathbf{j}) + (7\mathbf{i} + 5\mathbf{j}) = (2 + 7)\mathbf{i} + (-11 + 5)\mathbf{j} = 9\mathbf{i} - 6\mathbf{j}$ N

So the magnitude of $R = \sqrt{9^2 + (-6)^2} = \sqrt{81 + 36} = \sqrt{117} = 3\sqrt{13}$ N

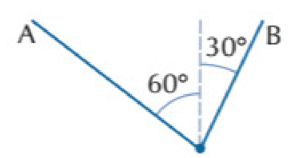
If you're asked for the direction, remember that it's usually measured anticlockwise from the positive horizontal direction.

Practice Questions

Q1 Find the magnitudes and directions (measured from the horizontal) of the resultant force in each situation.



Q2 A particle of weight W N is suspended in equilibrium by two light wires A and B, which make angles of 60° and 30° to the vertical respectively, as shown. The tension in A is 20 N. Find the tension in wire B and the value of W.



Exam Questions

Q1 A force of magnitude 7 N acts on a particle in the **i**-direction. Another force, of magnitude 4 N, acts on the particle with a direction of 30°. The resultant of these forces, R, has direction α.

Find: a) the magnitude of the force R, [3 marks]

b) the angle α. [2 marks]

Q2 A sledge is held at rest on a smooth slope angled at 25° to the horizontal. The rope holding the sledge is at an angle of 20° above the slope. The normal reaction acting on the sledge due to contact with the surface is 80 N.

Find: a) the tension, T, in the rope, [3 marks]

b) the weight of the sledge. [2 marks]

Take it from me — once you start making puns, it's a slippery slope...

Resolving forces is some pretty classic vectors stuff, but make sure those tricksy inclined planes don't catch you out.