More on Connected Particles

More complicated pulley and peg questions have friction for you to enjoy too. Truly, your cup runneth over.

Remember to use $F \leq \mu R$ (or $F = \mu R$) on **Rough Planes**

Example: The model in the example on page 209 is refined to include a frictional force from the table. The coefficient of friction between the 3 kg mass and the table, μ, is assumed to be 0.5. Find the new tension in the string when the particles are released from rest. Take $g = 9.8$ ms^{-2}.

For B: Resolving vertically: $5g - T = 5a$ ①

For A: Resolving horizontally: $F_{net} = ma$

$T - F = 3a$ ②

Resolving vertically: $R - 3g = 0$

$\mathbf{R = 3g}$

The particles are moving, so $F = \mu R = 0.5 \times 3g$

$\mathbf{F = 14.7\ N}$

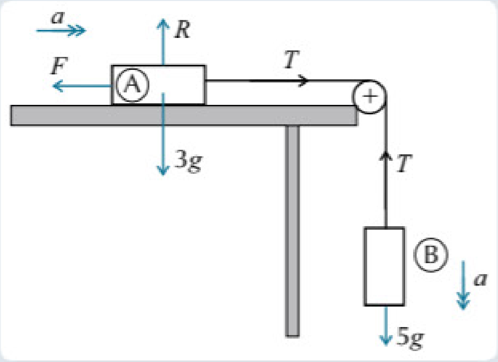
Sub this into ②: $T - 14.7 = 3a$

$a = \frac{1}{3}(T - 14.7)$

Sub this into ①: $5g - T = 5 \times \frac{1}{3}(T - 14.7)$

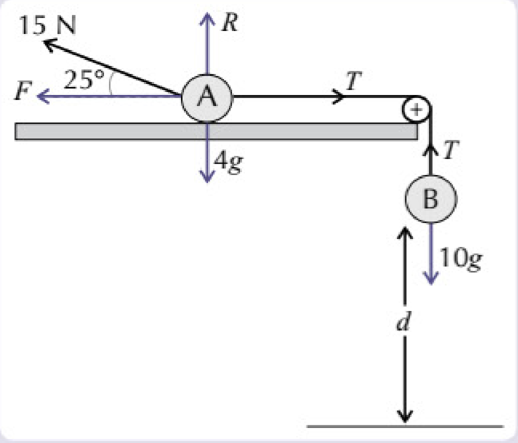
$8T = 147 + 73.5$

$\mathbf{T = 27.6\ N}$ (3 s.f.)



Resolve forces horizontally and vertically on each object and eventually you'll know everything there is to know about the system.

Example: Particles A and B of mass 4 kg and 10 kg respectively are connected by a light, inextensible string over a smooth pulley as shown. A force of 15 N acts on A at an angle of 25° to a rough horizontal plane where $\mu = 0.7$. When B is released from rest it takes 2 s to fall d m to the ground. Find d. Take $g = 9.8$ ms^{-2}.



For A: Resolve vertically to find R:

$R = 4g - 15 \sin 25° = 32.86...$ N

$F = \mu R = 0.7 \times 32.86... = 23.00...$ N

Now, resolving horizontally:

$T - 23.00... - 15 \cos 25° = 4a$

so $T = 4a + 36.59...$ ①

For B: Resolve vertically:

$10g - T = 10a$

so $T = 98 - 10a$ ②

Substitute ① into ②:
$4a + 36.59... = 98 - 10a$
so $a = 4.385...$ ms^{-2}

Using $s = ut + \frac{1}{2}at^2$: $s = d$, $u = 0$, $a = 4.385...$, $t = 2$

So $d = (0 \times 2) + \frac{1}{2}(4.385... \times 2^2) =$ **8.77 m** (3 s.f.)

When B hits the ground, A carries on moving along the plane. How long does it take A to stop after B hits the ground?

Speed of A when B hits the ground:

$v = u + at$: $u = 0$, $a = 4.385...$, $t = 2$

$v = 0 + (4.385... \times 2) = 8.771...$ ms^{-1}

Resolve to find stopping force on A:

$F_{net} = 23.00... + 15 \cos 25° = 36.59...$ N

Deceleration, $a = \frac{F_{net}}{m} = \frac{36.59...}{4} = 9.149...$ ms^{-2}

Time taken to stop:

$t = \frac{(v - u)}{a}$: $v = 8.771...$, $u = 0$, $a = 9.149...$

so $t = \frac{8.771...}{9.149...} =$ **0.959 s** (3 s.f.)

More on Connected Particles

Rough Inclined Plane *questions need* ***Really Good*** *force diagrams*

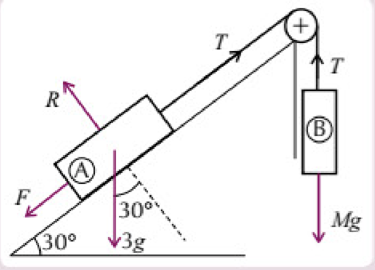
You know the routine... resolve forces parallel and perpendicular to the plane... *yawn*

Example: A 3 kg mass is held in equilibrium on a rough ($\mu = 0.4$) plane inclined at 30° to the horizontal. It is attached by a light, inextensible string to a mass of M kg hanging vertically beneath a smooth pulley, as shown in the diagram. Find M if the 3 kg mass is on the point of sliding up the plane. Take $g = 9.8 \text{ ms}^{-2}$.

For B: Resolving vertically: $F_{net} = ma$

$Mg - T = M \times 0$

$\boldsymbol{T = Mg}$



For A: Resolving perpendicular (↖): $F_{net} = ma$

$R - 3g \cos 30° = 3 \times 0$

$\boldsymbol{R = 3g \cos 30°}$

It's limiting friction, so: $F = \mu R = 0.4 \times 3g \cos 30°$

$\boldsymbol{F = 1.2g \cos 30°}$

For A: Resolving parallel (↗):

$T - F - 3g \sin 30° = 3 \times 0$

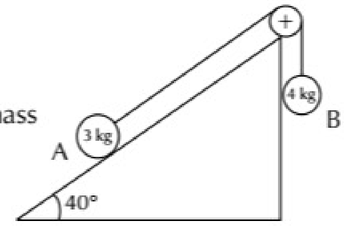
$Mg - 1.2g \cos 30° - 3g \sin 30° = 0$

$M - 1.039... - 1.5 = 0$

$\boldsymbol{M = 2.54 \text{ kg}}$ (3 s.f.)

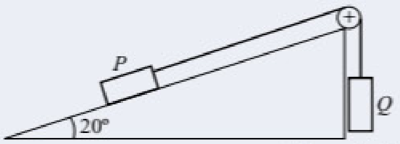
Practice Question

Q1 Two particles, A and B, of mass 3 kg and 4 kg respectively, are connected by a light, inextensible, string passing over a smooth pulley as shown. The 3 kg mass is on a smooth slope angled at 40° to the horizontal. Find the acceleration of the system if released from rest, and find the tension in the string. What force acting on the 3 kg mass parallel to the plane would be needed to maintain equilibrium?



Exam Questions

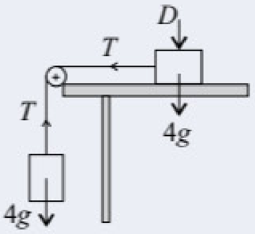
Q1 Two particles P and Q of masses 1 kg and M kg respectively are linked by a light inextensible string passing over a smooth pulley as shown. Particle P is on a rough slope inclined at 20° to the horizontal, where the coefficient of friction between P and the slope is 0.1.



a) Given that P is about to slide down the plane, find the mass of Q to 2 s.f. [5 marks]

b) When the mass of Q is 1 kg, it pulls P up the slope. Find the acceleration of P in this situation. [5 marks]

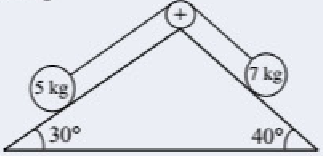
Q2 A box of mass 4 kg rests on a rough table, connected to an identical box, hanging over the edge of the table, by a light, inextensible string passing over a smooth pulley. The box is held in place by a force, D, acting perpendicular to the table as shown. The coefficient of friction between the box and the table is 0.6.



a) Find the minimum magnitude of D required for the boxes to remain at rest. [5 marks]

b) The force D is removed. Find the velocity of the boxes 2 seconds later (assuming that the boxes do not hit the floor or the pulley). [6 marks]

Q3 Two particles of mass 5 kg and 7 kg are connected by a light inextensible string passing over a smooth pulley as shown. Given that the coefficient of friction between each particle and the rough surface is 0.15, and that neither particle strikes the pulley, find the acceleration of the 5 kg particle. [8 marks]



Rough, inclined planes AND connected particles? You're pulley my leg...

The key word here is rough. If a question mentions the surface is rough, then cogs should whirr and the word 'friction' should pop into your head. Take your time with force diagrams of rough inclined planes — I had a friend who rushed into drawing a diagram, and he ended up with a broken arm. But that was years later, now that I come to think of it.