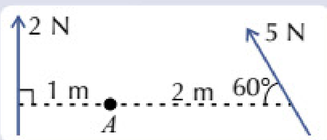
Moments

In this lifetime there are moments: moments of joy and of sorrow, and those moments where you have to answer questions on moments in exams. You might also want to take a moment to brush up on resolving forces.

*Moment = Force × **Perpendicular** Distance from the force's **Line of Action***

A '**moment**' is the **turning effect** a force has **around a point**. The **larger the force**, and the **greater the distance** from a point, then the **larger the moment**. In an exam question, you might be given a distance between the point and the force that's **not perpendicular** to the force's '**line of action**'. You'll need to **resolve** to find the **perpendicular distance** or the **perpendicular component** of the force.

Example: Find the sum of the moments of the forces shown about the point A.

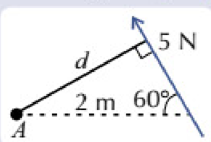


Calculating the **clockwise moment** is simple as the line of action is perpendicular to A:

$2 \times 1 =$ **2 Nm**

The units of moments are newton-metres (Nm) — unimaginative, but easy to remember.

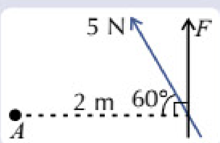
The **anticlockwise moment** is trickier as the line of action of the force **isn't perpendicular** to A. There are two ways to go about finding the moment — by finding the **perpendicular distance** or finding the **perpendicular component** of the force.



Finding the **perpendicular distance**:

$d = 2 \sin 60°$

So, moment $= 5 \times 2 \sin 60° = 10 \sin 60° =$ **$5\sqrt{3}$ Nm**



Finding the **perpendicular component** of the force:

$F = 5 \sin 60°$

So, moment $= 5 \sin 60° \times 2 = 10 \sin 60° =$ **$5\sqrt{3}$ Nm**

Both methods give the same moment. Just choose whichever you find simplest — and be sure to show your workings.

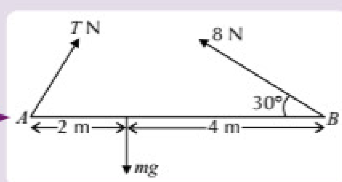
We can now find the **sum** of the moments (in this case, taking anticlockwise as negative):

Clockwise + anticlockwise moments $= 2 + (-5\sqrt{3}) = -6.6602...$ Nm = **6.66 Nm anticlockwise** (3 s.f.)

If a system is in **equilibrium**, **anticlockwise moments** = **clockwise moments** about **any** point.

Example: A rod, AB, of length 6 m is held in **equilibrium** by two strings, as shown. By taking moments, find the mass, m, of the rod. Take $g = 9.8$ ms^{-2}.

Although you can take moments about any point, it's usually easier to take moments about a point that has an unknown force going through it. Here, the moment of T about A = 0.



By taking moments **about A**:

clockwise moments = anticlockwise moments

$2mg = 6 \sin 30° \times 8$

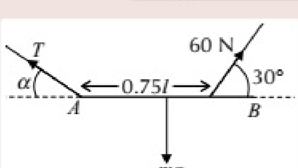
$mg = 12$

$m = 12 \div 9.8 =$ **1.22 kg** (3 s.f.)

*The **Weight** acts at the **Centre** of a **Uniform** rod*

A model **rod** has **negligible thickness**, so you only need to consider where along its **length** the centre of mass lies. If the rod is **uniform** then the weight acts at the **centre** of the rod.

Example: A uniform rod, AB, of length l m and mass m kg is suspended horizontally in equilibrium by two inextensible wires, with tensions as shown. Find m.



Taking moments about A:

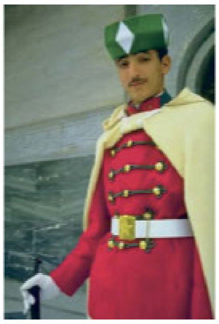
$mg \times 0.5l = 60 \sin 30° \times 0.75l$

$mg \times 0.5 = 30 \times 0.75$

$4.9m = 22.5$

m = 4.59 kg (3 s.f.)

Again, you can pick any point to take moments about, but it makes sense to choose A, because that eliminates the unknown force, T.



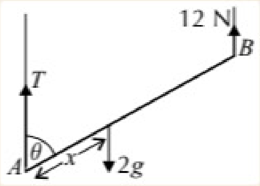
If this chap was called Rod, this caption would've been hilarious...

Moments

You can calculate the Centre of Mass for Non-Uniform rods

If the weight acts at an **unknown** point along a rod, the point can be found in the usual way — by taking **moments**. You might also have to **resolve** the forces **horizontally** or **vertically** to find some missing information.

Example: A non-uniform rod, AB, of mass 2 kg and length 1 m, is suspended in equilibrium at an angle of θ to the vertical by two vertical strings, as shown. The tensions in the strings are T N and 12 N respectively. Find the distance, x, from A to the rod's centre of mass.



Taking moments about A: clockwise moments = anticlockwise moments

$$2g \sin\theta \times x = 12 \sin\theta \times 1$$

2 sin θ cancels

$$gx = 6 \Rightarrow x = 0.612 \text{ m (3 s.f.)}$$

The 'Point of Tilting' means Other Reactions are Zero

If a rod is **resting** on a **support**, then there will be a **normal reaction** force acting on the rod.
If it's '**on the point of tilting**' or '**about to tilt**' about a particular support, then any normal reactions acting at any **other** supports along the rod will be zero. This works exactly the same way for **tension** in **strings** supporting the rod (like in the previous example). This should make sense in the context, when you think about which way the rod is about to rotate. You can usually assume that supports and strings are **fixed**, and won't move with the rod.

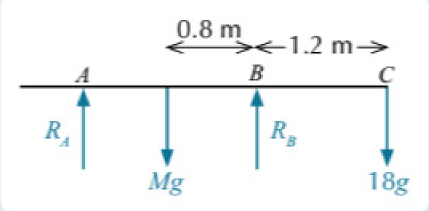
Example: A non-uniform wooden plank of mass M kg rests horizontally on supports at A and B, as shown. When a bucket of water of mass 18 kg is placed at point C, the plank is in equilibrium, and is on the point of tilting about B. Find the value of M and the magnitude of the reaction at B.

1) Taking moments about B:

$$(18g \times 1.2) + 0 = Mg \times 0.8$$

The plank is on the point of tilting about B, so $R_A = 0$ (the moment is also zero).

$$\Rightarrow M = (18 \times 9.8 \times 1.2) \div (9.8 \times 0.8) = \mathbf{27\ kg}$$



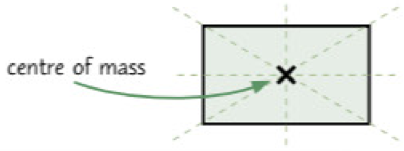
2) Resolving vertically:

$$R_A + R_B = Mg + 18g$$

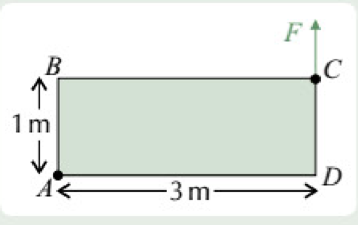
$$0 + R_B = 27g + 18g \Rightarrow R_B = \mathbf{441\ N}$$

Laminas are Two-Dimensional objects

A **lamina** is a flat 2D object (its thickness can be ignored). The centre of mass of a **uniform rectangular** lamina is at the symmetrical centre of the rectangle.



Example: A uniform rectangular lamina, $ABCD$, of weight 8 N, is pivoted at point A. The lamina is held in equilibrium by a vertical force F acting at point C, as shown.
a) Find the horizontal and vertical distances of the centre of mass of the lamina from A.
b) Find the magnitude of the force F.



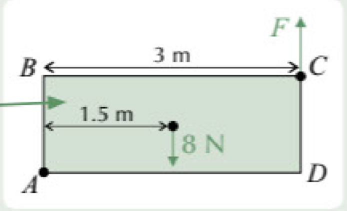
a) The centre of mass is at the centre of the lamina, so the horizontal distance is $3 \div 2 = \mathbf{1.5\ m}$ and the vertical distance is $1 \div 2 = \mathbf{0.5\ m}$

You'll need to know the position of the centre of mass to figure out the perpendicular distance of the weight force from the pivot.

b) Both the lamina's weight and F act vertically, so the perpendicular distance from each force to the pivot is just the horizontal distance.

Taking moments about A: $8 \times 1.5 = F \times 3$

$$\Rightarrow F = 12 \div 3 = \mathbf{4\ N}$$

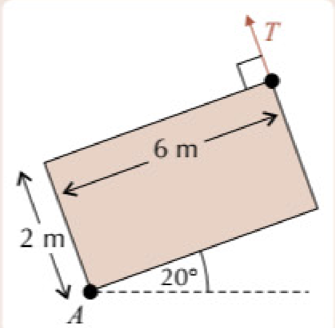


Moments

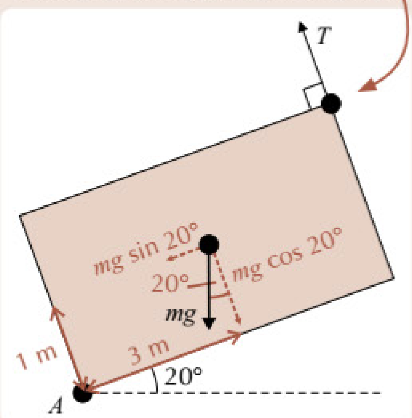
Resolve Forces relative to the Edges of the Lamina

If there are forces acting **at angles** to the sides of the lamina, it's usually easier to resolve them **perpendicular** and **parallel** to the lamina's sides. But remember that **both components** will have a turning effect on the lamina.

Example: A uniform rectangular lamina of mass 12 kg is pivoted at one corner at A. A light, inextensible string attached at the opposite corner applies a tension force T to the lamina, as shown.
Given that the string holds the lamina in equilibrium at an angle of 20° to the horizontal, find the tension in the string.



1) Here, the lamina's weight acts at 20° to its sides. Find the components of its weight parallel and perpendicular to the sides of the lamina. Drawing a diagram will be helpful:



2) Now you can take moments about A, but watch out — the two components of the weight force **both** have turning effects, one clockwise and one anticlockwise. They also have **different perpendicular distances**, so don't get caught out.

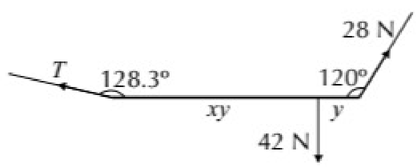
3) Taking moments about A:

clockwise moments = anticlockwise moments

$$(mg \cos 20° \times 3) = (mg \sin 20° \times 1) + (T \times 6)$$
$$6T = mg(3 \cos 20° - \sin 20°)$$
$$T = 19.6(2.4770...)$$
$$= \mathbf{48.6\ N}\ (3\text{ s.f.})$$

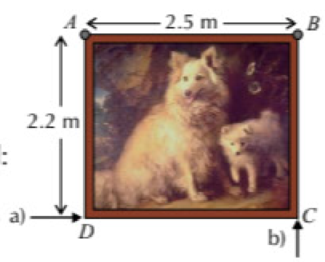
You could also do this without breaking the weight into components — instead, you'd find the horizontal distance from A to the COM, which is the perpendicular distance from A to the line of action of the weight.

Practice Questions



Q1 Given that the rod in the diagram is in equilibrium, calculate the value of x and the magnitude of T.

Q2 A painting of mass 10 kg, modelled as a uniform rectangular lamina $ABCD$, hangs from two fixed, smooth pegs at A and B, as shown. The peg at B breaks. Calculate the force required to hold the painting in equilibrium, if the force is applied:
a) horizontally at D, b) vertically at C.



Exam Questions

Q1 A uniform rod AB of mass 7 kg is suspended horizontally in equilibrium from two fixed inextensible strings. One is attached at P, 1 m from A, and the other is at Q, 2 m from B. Given that the length of the rod is 9 m, find:

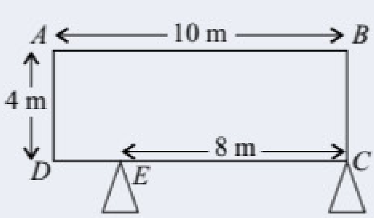
a) the tension in each string, T_P and T_Q, [4 marks]

b) the maximum mass, M, that could be attached to the rod without causing it to tilt, if the mass were attached at: (i) A, (ii) B. [6 marks]

Q2 A uniform rectangular lamina, $ABCD$, of mass 20 kg, rests on two rough supports: one at C, and another at E, 8 m from C as shown.

a) Calculate the reaction forces from the supports at C and E. [4 marks]

b) A horizontal force F is applied at B, acting towards A. Given that the lamina is now on the point of tilting about E, calculate the magnitude of the force F. [3 marks]



Resolve the force, Luke — or use the perpendicular distance...

The only tricky thing here is deciding which point to take moments about. Once you've done loads of these questions, you'll find you develop a soft spot for them. Don't believe me? Search your feelings — you know it to be true.