Summary of key points

1 **Newton's first law** of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.

2 A **resultant** force acting on an object will cause the object to **accelerate in the same direction** as the resultant force.

3 You can find the **resultant** of two or more forces given as vectors by adding the vectors.

4 **Newton's second law** of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.

5 $W = mg$

6 You can use $\mathbf{F} = m\mathbf{a}$ to solve problems involving vector forces acting on particles.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Forces

A force acting on an object has **direction** and **magnitude**. The units of force are **newtons** (N). 1 newton is the force needed to accelerate a 1 kg object at a rate of $1\,\text{m s}^{-2}$. Because of this, the units of force can be written as kg m s^{-2}.

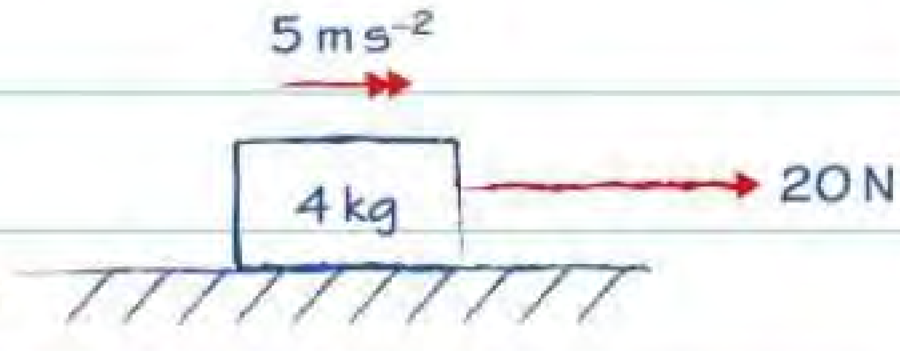
$F = ma$

$F = ma$ is sometimes called the **equation of motion**. In words it is:

force (N) = mass (kg) × acceleration (m s^{-2})

You need to remember $F = ma$. It is not in the formulae booklet.

This 4 kg block is resting on a smooth surface. If it is acted on by a force of 20 N it will accelerate at a rate of $5\,\text{m s}^{-2}$.



Resultant force

If there is more than one force acting on a particle you can find the **resultant** in any given direction.

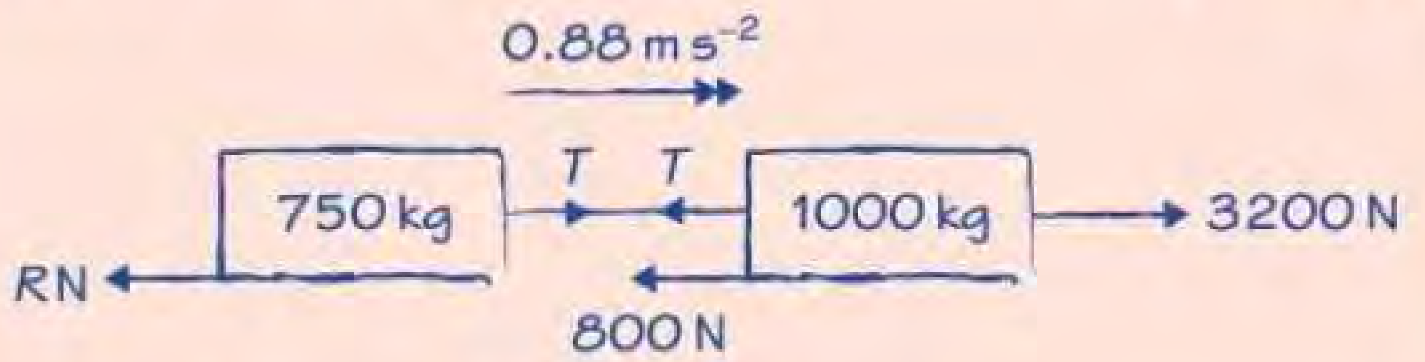
This boat is accelerating. The vertical forces have the same magnitude so their resultant is **zero**. The resultant force in the horizontal direction is 5500 − 1000 = 4500 N.

Worked example

A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is $0.88\,\text{m s}^{-2}$,

(a) show that $R = 860$ **(3 marks)**



Drawing a large, well-labelled diagram will help you see what is going on. For part **(a)** consider the caravan and the car as a **single system**. The resultant force acting on the system is $3200 - 800 - R$, and the combined mass is $750 + 1000$.

Using $F = ma$ for the whole system:

$$3200 - 800 - R = (750 + 1000) \times 0.88$$
$$2400 - R = 1540$$
$$R = 860\text{ N}$$

(b) find the tension in the tow-bar. **(3 marks)**

Using $F = ma$ for the caravan only:

$$T - R = 750 \times 0.88$$
$$T - 860 = 660$$
$$T = 1520\text{ N}$$

For part **(b)** you need to consider either the caravan or the car on its own.

Tension and thrust

- **Tension** is a force which will tend to **stretch** a rod, spring or string.
- **Thrust** is a force which will tend to **compress** a rod.

When a car accelerates it produces a tension in the tow-bar which in turn accelerates the caravan. If the car brakes, it produces a thrust in the tow-bar which decelerates the caravan.

Now try this

A car of mass 1200 kg pulls a trailer of mass 400 kg along a straight horizontal road using a light tow-bar. The resistances to motion of the car and trailer have magnitudes 500 N and 200 N respectively. The engine of the car produces a constant driving force of magnitude 1500 N. Find

(a) the acceleration of the car and trailer Answer **(3 marks)**

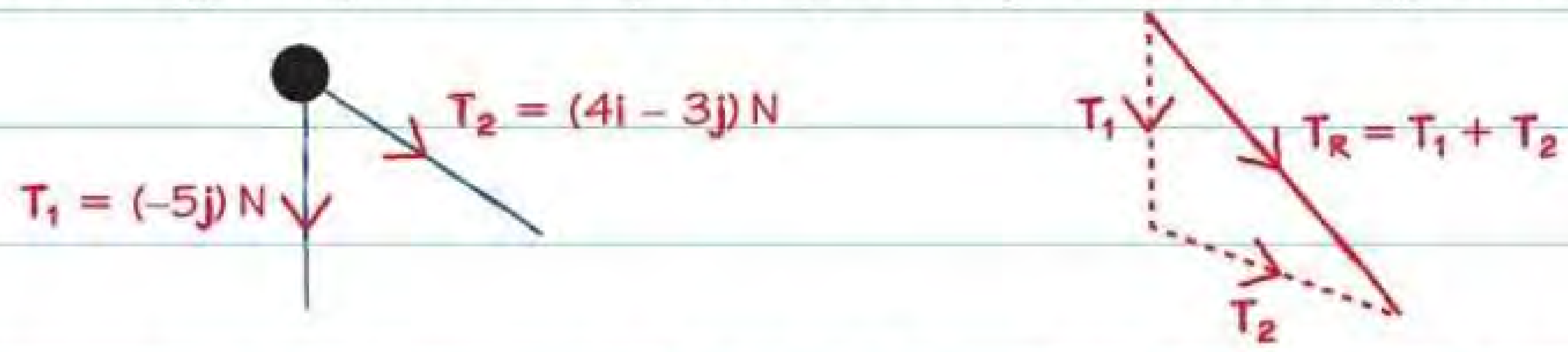
(b) the magnitude of the tension in the tow-bar. Answer **(3 marks)**

Forces as vectors

Force is a **vector quantity.** This means it has **direction** as well as **magnitude.** You can represent forces as **column vectors** or using **i-j notation**. Have a look at page 33 for a reminder about these.

Finding resultants

To find the **resultant** of forces given as vectors, you need to **add** the vectors. You can add the **i** components and add the **j** components separately. This particle is being acted on by two forces.



The resultant force acting on the particle is $\mathbf{T_R} = \mathbf{T_1} + \mathbf{T_2} = (-5\mathbf{j}) + (4\mathbf{i} - 3\mathbf{j}) = (4\mathbf{i} - 8\mathbf{j})$ N

So it has magnitude $|\mathbf{T_R}| = \sqrt{4^2 + 8^2} = 8.94$ N

Worked example

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})$ N $\quad \mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})$ N $\quad \mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q. **(3 marks)**

$\underset{\sim}{F_1} + \underset{\sim}{F_2} + \underset{\sim}{F_3} = 0$

$(7\underset{\sim}{i} - 9\underset{\sim}{j}) + (5\underset{\sim}{i} + 6\underset{\sim}{j}) + (p\underset{\sim}{i} + q\underset{\sim}{j}) = 0$

$\underset{\sim}{i}$ components: $7 + 5 + p = 0$, so $p = -12$

$\underset{\sim}{j}$ components: $-9 + 6 + q = 0$, so $q = 3$

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Find

(b) the magnitude of $\mathbf{R}$ **(2 marks)**

$\underset{\sim}{R} = \underset{\sim}{F_1} + \underset{\sim}{F_2}$

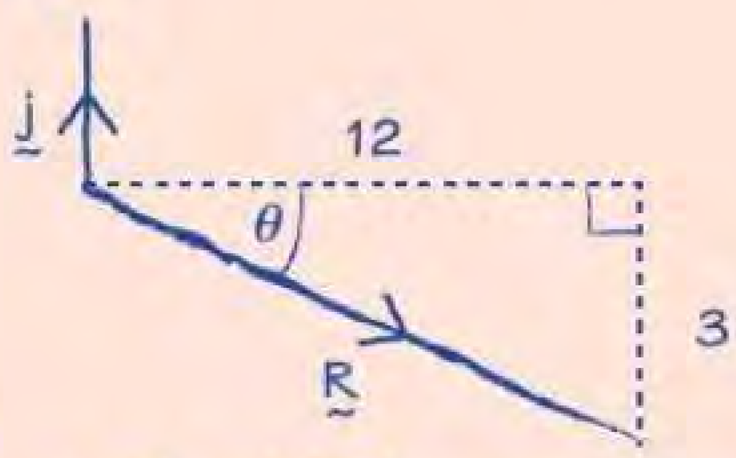
$= (7\underset{\sim}{i} - 9\underset{\sim}{j}) + (5\underset{\sim}{i} + 6\underset{\sim}{j})$

$= (12\underset{\sim}{i} - 3\underset{\sim}{j})$ N

$|\underset{\sim}{R}| = \sqrt{12^2 + (-3)^2} = 12.4$ N (3 s.f.)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$. **(3 marks)**

$\underset{\sim}{R} = (12\underset{\sim}{i} - 3\underset{\sim}{j})$ N



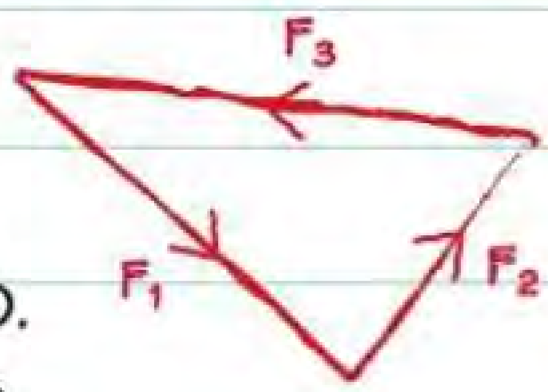
$\tan\theta = \frac{3}{12}$, so $\theta = 14°$ (nearest degree)

So the angle between $\underset{\sim}{j}$ and $\underset{\sim}{R}$ is $90° + 14° = 104°$

For part (c), draw a sketch to make sure you calculate the correct angle.

Equilibrium

If a particle is in equilibrium, the resultant force acting on it is ZERO. The vectors of any forces acting on it will form a closed **triangle**.



For forces acting on a particle in equilibrium:

- ✓ the **i** components add up to zero
- ✓ the **j** components add up to zero.

Be careful with **accuracy** in vectors questions.

- Give **vectors** exactly in terms of **i** and **j**.
- Give **magnitudes** as surds, or to 3 s.f.
- Give **angles** correct to the **nearest degree**.

Now try this

A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by $\mathbf{F}_1 = (2\mathbf{i} - 6\mathbf{j})$ N and $\mathbf{F}_2 = (3\mathbf{i} + k\mathbf{j})$ N.

Given that the resultant $\mathbf{R}$ of the two forces acts in a direction parallel to the vector $(\mathbf{i} - \mathbf{j})$,

(a) find the value of k. Answer **(4 marks)**

A third force $\mathbf{F}_3$, given by $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N is applied to the particle, so that it now lies in equilibrium.

(b) Find the value of p and the value of q.

Answer **(3 marks)**

Motion in 2D

Acceleration is a vector quantity that can be written as a column vector or using **i-j** notation. You can use this to describe motion in two dimensions.

Equation of motion

You can write equations of motion involving forces and accelerations written as vectors:

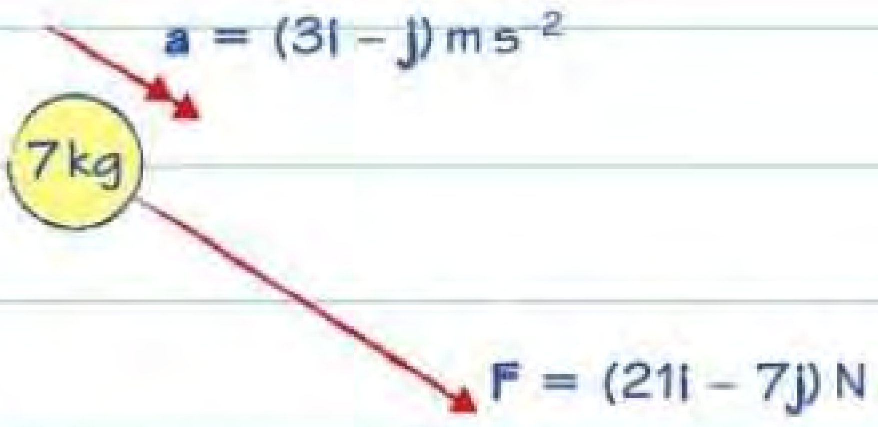
$$\mathbf{F} = m\mathbf{a}$$

F is the force vector in newtons

a is the acceleration vector in m s^{-2}

m is the mass in kg.

Mass is a **scalar** quantity. It has **magnitude** but **no direction**. This means that the direction of the force is the same as the direction of the acceleration.



This particle is accelerating from rest at a rate of **a** = (3**i** − **j**) m s^{-2}. This acceleration is produced by a force of **F** = 7(3**i** − **j**) = (21**i** − 7**j**) N.

Worked example

A hockey ball of mass 160 g is at rest on a smooth flat field. It is pushed by a hockey stick with a constant force of (–24**i** + 45**j**) N.

(a) Find the acceleration of the hockey ball. **(1 mark)**

$$\underset{\sim}{F} = m\underset{\sim}{a}$$

$$(-24\underset{\sim}{i} + 45\underset{\sim}{j}) = 0.16\underset{\sim}{a}$$

$$\underset{\sim}{a} = \frac{1}{0.16}(-24\underset{\sim}{i} + 45\underset{\sim}{j})$$

$$= (-150\underset{\sim}{i} + 281.25\underset{\sim}{j})\ \text{m s}^{-2}$$

(b) Given that the ball leaves the hockey stick with a velocity of 44 m s^{-1}, find the length of time that the stick is in contact with the ball. **(3 marks)**

$$|a| = \sqrt{150^2 + 281.25^2} = 318.75\ \text{m s}^{-2}$$

$$v = u + at$$

$$44 = 0 + 318.75t$$

$$t = 0.138\ \text{s (3 s.f.)}$$

(c) Criticise this model in respect of:

(i) the field

(ii) the motion of the ball. **(2 marks)**

(i) A hockey field is unlikely to be smooth so there would be some resistance due to friction.

(ii) The force applied by the hockey stick is unlikely to be constant.

You have been given the force as a vector, so you need to give the acceleration in vector form. Use the vector equation of motion **F** = m**a**.

In books vectors are often written in bold type. In your exam you can use a wiggly line underneath the letters to denote a vector quantity.

The ball starts **at rest** and the force acts in a straight line, so the ball will accelerate uniformly in a straight line. You can use the ***suvat*** equations, taking the **magnitude** of the acceleration as a.

Criticising models

You might be asked to criticise or refine a model in your exam. You should think about the real-life situation and compare this to the model. Try to identify elements of the model that are unrealistic. In real life:

- ✓ surfaces are rarely smooth
- ✓ forces and accelerations are rarely constant
- ✓ objects have dimension and are subject to air resistance and rotation
- ✓ strings and rods may deform.

Now try this

1 A force of $\begin{pmatrix} 24 \\ -10 \end{pmatrix}$ N acts on a particle of mass 8 kg.

(a) Find the acceleration of the particle, giving your answer as a column vector. **(1 mark)**

Answer

(b) Find the magnitude and bearing of the acceleration of the particle. **(4 marks)**

Answer

2 A particle of mass 0.6 kg is being acted on by three forces $\mathbf{F}_1$N, $\mathbf{F}_2$N and $\mathbf{F}_3$N, where

$\mathbf{F}_1 = (7\mathbf{i} - 2\mathbf{j})$ $\mathbf{F}_2 = (p\mathbf{i} - \mathbf{j})$ $\mathbf{F}_3 = q\mathbf{j}$

Given that the particle is accelerating at $(3\mathbf{i} + 6\mathbf{j})$ m s^{-2}, find the value of p and the value of q. **(3 marks)**

Answer

Motion under gravity

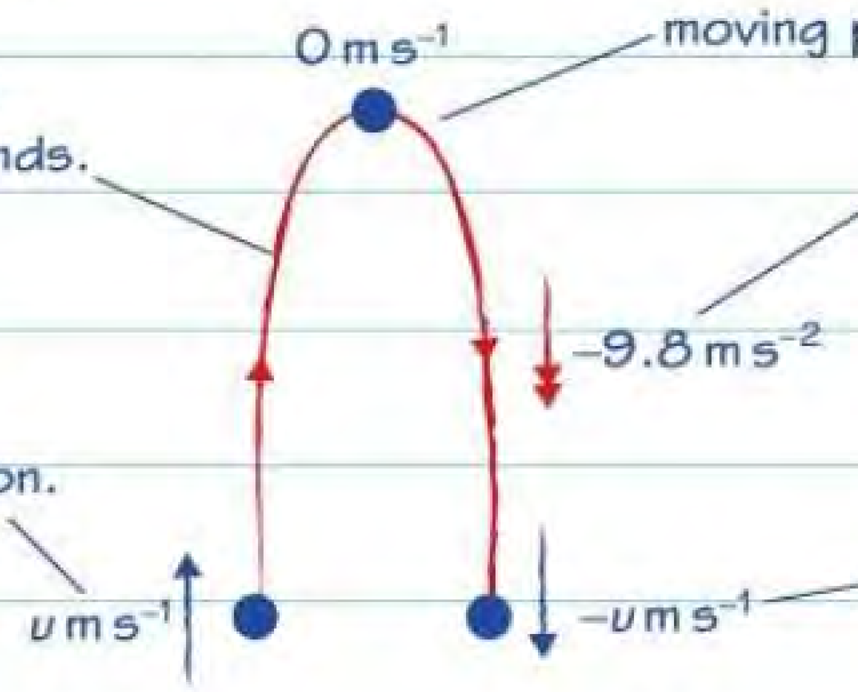
If you ignore **air resistance**, you can model an object moving freely under gravity as a particle with **constant downward acceleration**. This means you can use the constant acceleration formulae given on pages 82 and 83.

Up and down

Here are some useful facts about particles moving vertically under gravity.

If the particle is thrown upwards and takes t seconds to reach its **maximum height** then it returns to its starting position after $2t$ seconds.

You can choose which direction is positive in a question. You usually want UP to be the positive direction.



At its **maximum height**, a vertically moving particle has **zero** velocity.

UP is the positive direction, so the acceleration due to gravity is **negative**. It remains **constant** throughout. Show acceleration with a double arrow so you don't confuse it with velocity.

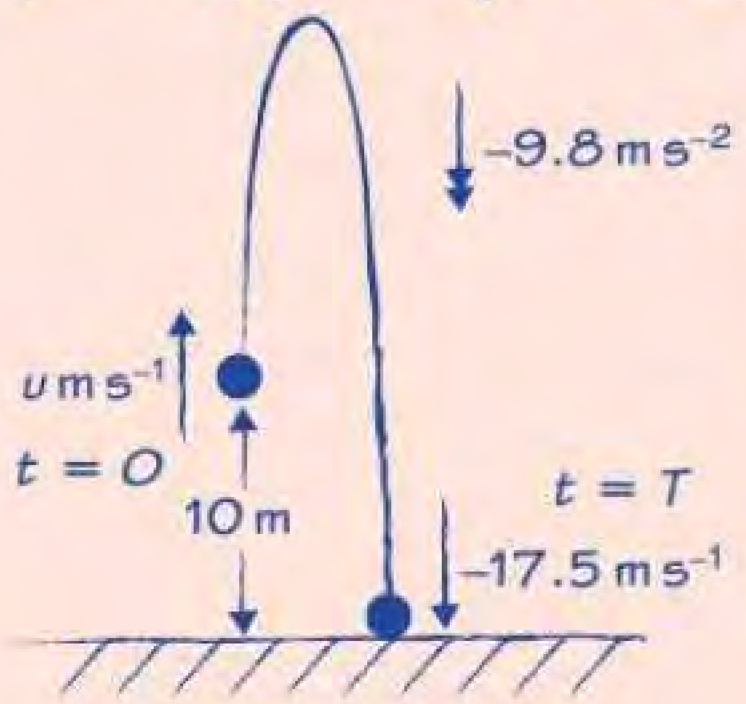
When the particle returns to its initial position, it has the **same speed** as it started with but in the **opposite direction**.

Worked example

At time $t = 0$, a particle is projected vertically upwards with speed $u\,\mathrm{m\,s^{-1}}$ from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed $17.5\,\mathrm{m\,s^{-1}}$. Find

(a) the value of u **(3 marks)**

$s = -10$, $u = ?$, $v = -17.5$, $a = -9.8$, ~~$t = T$~~



The **positive** direction is **up**, so a and v are **negative**. The finishing position is 10 m **below** the starting position, so s is negative as well.

$v^2 = u^2 + 2as$

$(-17.5)^2 = u^2 + 2 \times (-9.8) \times (-10)$

$306.25 = u^2 + 196$

$u^2 = 110.25$

$u = 10.5 = 11\,\mathrm{m\,s^{-1}}$ (2 s.f.)

(b) the value of T. **(4 marks)**

~~$s = -10$~~, $u = 10.5$, $v = -17.5$, $a = -9.8$, $t = T$

$v = u + at$

$-17.5 = 10.5 + (-9.8) \times T$

$9.8T = 28$

$T = 2.857... = 2.9\,\mathrm{s}$ (2 s.f.)

You've used $g = 9.8\,\mathrm{m\,s^{-2}}$ so round to 2 s.f.

Accuracy and gravity

Unless you're told otherwise, you should use this value for the acceleration due to gravity:

$$g = 9.8\,\mathrm{m\,s^{-2}}$$

This value for g is correct to **2 significant figures**.

This means that if you use $g = 9.8\,\mathrm{m\,s^{-2}}$ in your calculation you should give your final answer correct to 2 significant figures.

Don't use rounded values in calculations. In the Worked example on the left, you can give the answer to part (a) as $11\,\mathrm{m\,s^{-1}}$, but you still need to use $u = 10.5$ in part (b).

Now try this

A diver projects herself upwards from a diving platform with a speed of $3.5\,\mathrm{m\,s^{-1}}$. The platform is 15 m above the water. Modelling the diver as a particle moving freely under gravity, find

(a) the greatest height above the water reached by the diver Answer **(4 marks)**

(b) the speed with which the diver hits the water **(3 marks)** Answer

(c) the total time from when the diver leaves the platform to when she hits the water. **(3 marks)**

Answer