Summary of key points

7 You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.

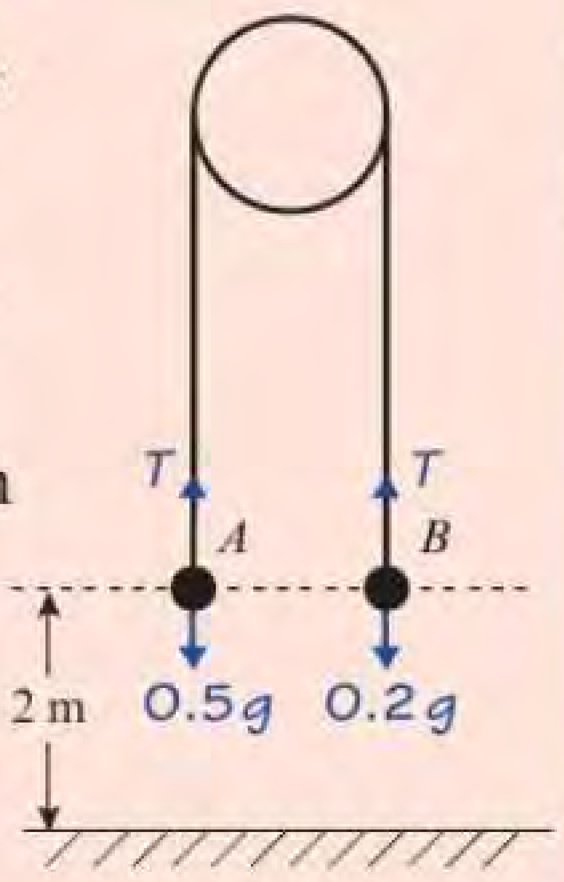
8 **Newton's third law** states that for every action there is an equal and opposite reaction.

Pulleys

A pulley is used to connect two particles. The particles interact through the **tension** in the string.

Worked example

Two particles A and B have masses 0.5 kg and 0.2 kg respectively. The particles are attached to the ends of a light inextensible string which passes over a smooth pulley. Both particles are held, with the string taut, at a height of 2 m above the floor. The particles are released from rest and in the subsequent motion B does not reach the pulley. Find



(a) the tension in the string immediately after the particles are released **(6 marks)**

$R(\downarrow)$: Using $F = ma$ for particle A:

$0.5g - T = 0.5a$ ①

$R(\uparrow)$: Using $F = ma$ for particle B:

$T - 0.2g = 0.2a$ ②

$2 \times$ ①: $g - 2T = a$

$-5 \times$ ②: $5T - g = a$

$2g - 7T = 0$

$T = \frac{2g}{7} = 2.8\,\text{N}$

(b) the acceleration of A immediately after the particles are released **(2 marks)**

Substitute $T = 2.8\,\text{N}$ into ①:

$0.5g - 2.8 = 0.5a$ so $a = 4.2\,\text{m s}^{-2}$

(c) the speed of A when it hits the ground. **(3 marks)**

$s = 2,\ u = 0,\ v = ?,\ a = \frac{3g}{7},\ \cancel{t = ?}$

$v^2 = u^2 + 2as$

$= 0^2 + 2 \times \frac{3g}{7} \times 2 = \frac{12g}{7}$

$v = 4.0987... = 4.1\,\text{m s}^{-1}$ (2 s.f.)

Using $F = ma$

When two particles are connected via a pulley, you will often have to write **two equations of motion** using $F = ma$.
You can solve these **simultaneously** to find any unknown values.

The tension in the string is the same at A as it is at B because the pulley is **smooth**. And both particles accelerate at the same rate, because the string is **inextensible**. There is more on **modelling assumptions** like this on page 80.

Problem solved!

Follow these steps for parts (a) and (b).

1. Label your diagram with the tension in the string, and the weight of both particles.
2. Write an equation of motion for each particle. You can resolve up or down for each particle, but remember that the acceleration acts in the opposite direction for B as it does for A.
3. Solve your two equations of motion simultaneously to find T and a.

You will need to use problem-solving skills throughout your exam – **be prepared!**

For part (c), you have to use the formulae for constant acceleration. Have a look at pages 82 and 83 for a reminder.

After the string is cut, particle B behaves as an object moving freely under gravity. Have a look at page 84 for more on this.

Now try this

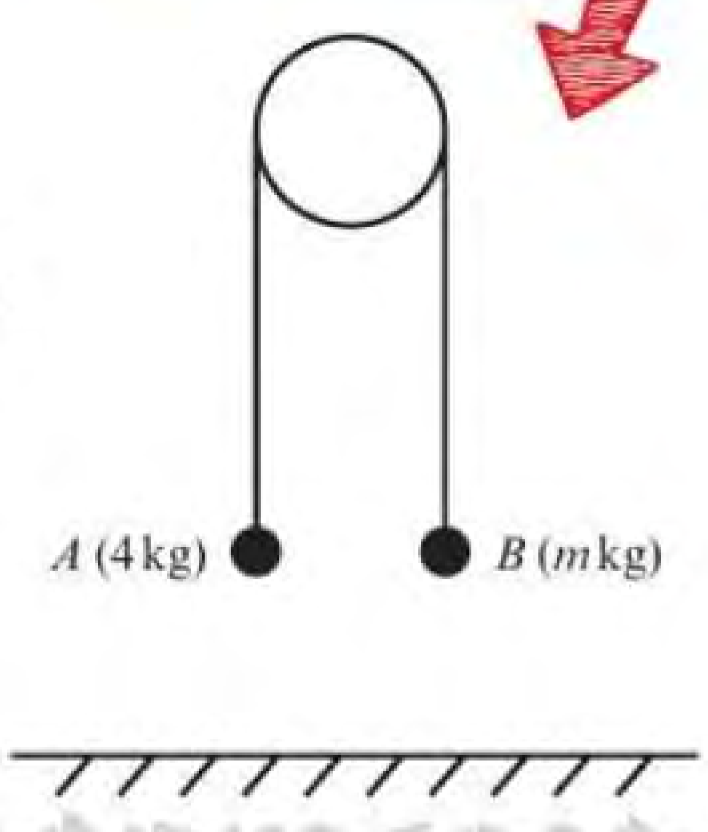
Two particles A and B have masses 4 kg and m kg respectively. They are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is released from rest, and A descends with acceleration $0.2g$.

(a) Find the tension in the string as A descends. Answer **(3 marks)**

(b) Find the value of m. Answer **(3 marks)**

After 1 s, the string is cut and particle B moves vertically under gravity.

(c) Find the time after the string is cut at which particle B returns to its initial position. Answer **(9 marks)**

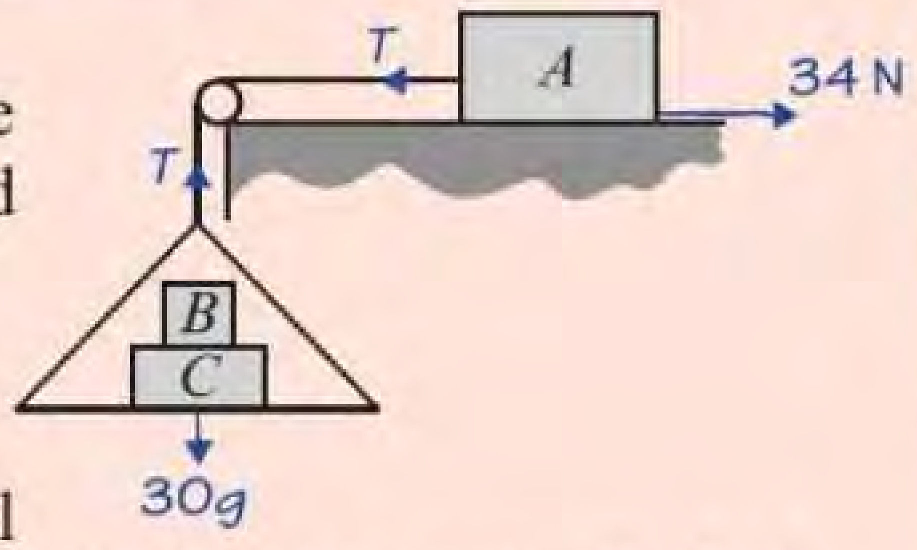


Connected particles

When you are solving problems involving connected objects, you need to decide when to consider the **whole system** and when to consider each object **individually**.

Worked example

One end of a light inextensible string is attached to a block A of mass 20 kg, which is on a rough horizontal table. The other end of the string is attached via a pulley to a light scale pan which carries two blocks B and C. The mass of block B is 8 kg and the mass of block C is 22 kg. The system is released from rest.



The resistance to motion of A from the rough table has constant magnitude 34 N. Find:

(a) (i) the acceleration of the scale pan

(ii) the tension in the string. **(8 marks)**

(i) Scale pan: $30g - T = 30a$ ①

Block A: $T - 34 = 20a$ ②

① + ②: $30g - 34 = 50a$

$260 = 50a$

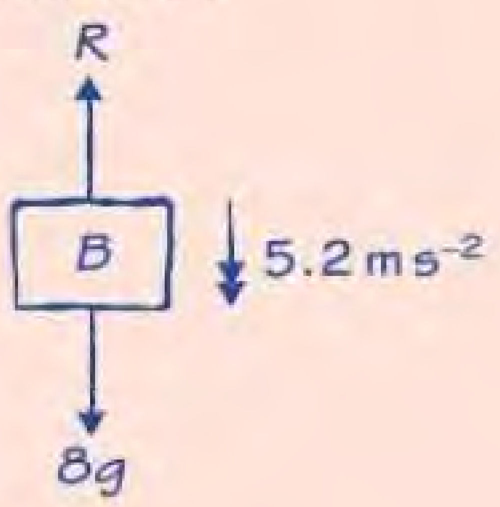
$a = 5.2\,\text{m s}^{-2}$

(ii) Substitute into ②:

$T - 34 = 20 \times 5.2$

$T = 138 = 140$ N (2 s.f.)

(b) the magnitude of the force exerted on block B by block C. **(3 marks)**



$8g - R = 8 \times 5.2$

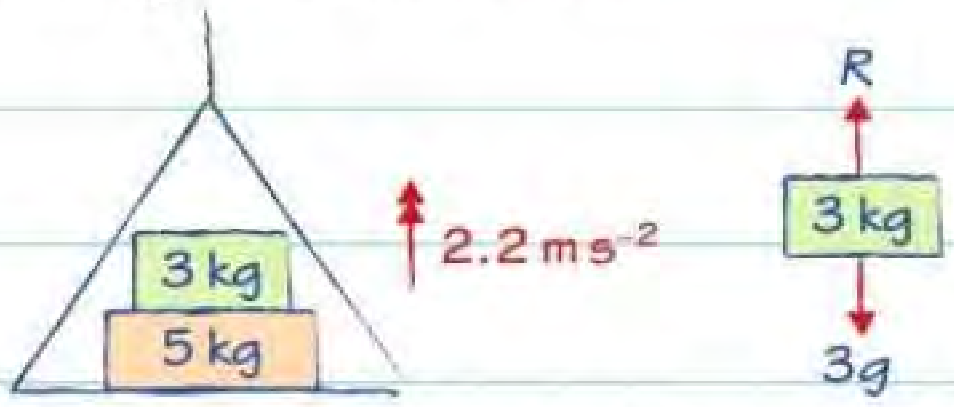
$R = 36.8$ N

The force exerted on B by C is 37 N (2 s.f.).

If you have to answer questions about **rough surfaces** in your AS exam the magnitude of the frictional force will be constant. It always acts so as to **oppose** the direction of motion.

Scale pans

If one block sits on top of another in a scale pan, you can find the **force exerted** by one block on the other. This scale pan is accelerating at $2.2\,\text{m s}^{-2}$.



On its own, the green block has **two** forces acting on it: its weight, and the **reaction** exerted on it by the orange block. An equation of motion for the green block (resolving in the upwards direction) is

$$R - 3g = 3 \times 2.2$$

So $R = 36$ N. Because the two blocks are not moving relative to each other they exert **equal** and **opposite** forces on each other, so the force exerted by the green block on the orange block is the same: 36 N.

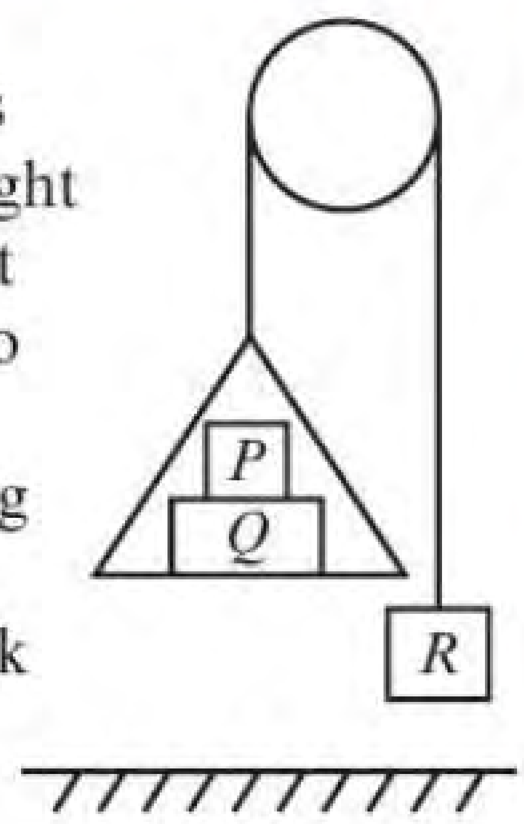
Problem solved!

Write separate equations of motion for A and for the whole scale pan, then solve simultaneously to find the tension and the acceleration. For part (b) you need to consider either block B or block C on its own.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

A block R of mass 0.8 kg is connected by means of a light inextensible string to a light scale pan, which carries two blocks P and Q. P and Q have masses 0.2 kg and m kg respectively. The system is released from rest and block R accelerates upwards at a rate of $1.5\,\text{m s}^{-2}$. Find



(a) the value of m Answer **(6 marks)**

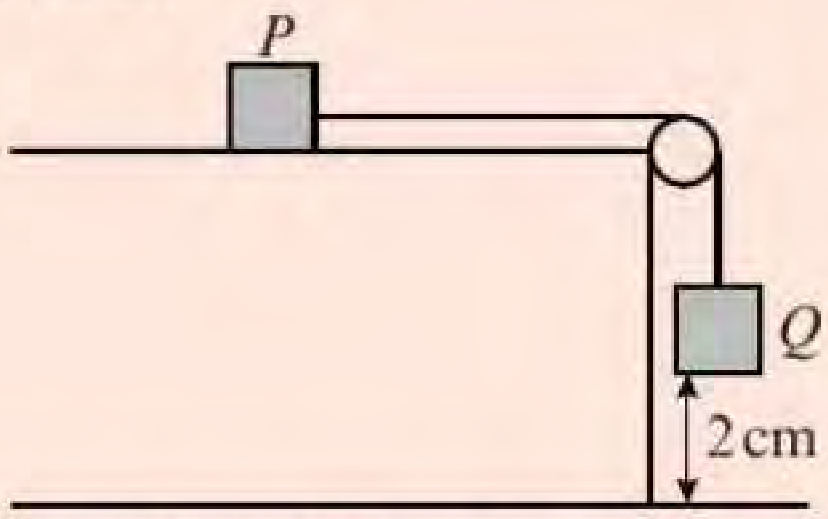
(b) the magnitude of the force exerted on block P by block Q. Answer **(3 marks)**

Combining techniques

You might need to answer questions involving forces, and *suvat* equations.

Worked example

A box, P, of mass 8 kg is held at rest on a rough platform. It is attached to another box, Q, of unknown mass by means of a light inextensible string that runs over a smooth pulley. Box Q hangs freely a distance of 2 m above a horizontal floor.



The system is released from rest with the string taut. The frictional force between P and the table is modelled as a constant force of magnitude 18.5 N and the boxes are modelled as particles.

(a) Given that box Q hits the floor after 2.5 seconds, use the model to find the mass of box Q. **(8 marks)**

$s = 2$, $u = 0$, $\cancel{v = ?}$, $a = ?$, $t = 2.5$

$s = ut + \frac{1}{2}at^2$

$2 = 0 + \frac{1}{2}a \times 2.5^2$

$a = 0.64 \text{ ms}^{-2}$

Let mass of Q be m kg and tension in string be T.

Equations of motion:

Q: $mg - T = m \times 0.64$ ①

P: $T - 18.5 = 8 \times 0.64$ ②

Solving ① and ② simultaneously:

$mg - 18.5 = 0.64m + 5.12$

$$m = \frac{5.12 + 18.5}{9.8 - 0.64}$$

$= 2.6$ kg (2 s.f.)

Following the experiment, box Q is weighed and is found to have a mass of 3 kg.

(b) In light of this information:

(i) comment on the validity of using this model to find the mass of box Q

(ii) suggest one possible improvement to the model. **(2 marks)**

(i) The model is not valid. The actual mass of box Q was larger. This could be because in reality the pulley is not smooth, or because air resistance was not considered.

(ii) The resistances to motion could be modelled as a variable force.

Golden rule

If the resultant force acting on a particle is constant, then its acceleration will be constant. This means you can apply the *suvat* formulae to the motion of the particle.

You know the time taken for box Q to hit the floor and the distance fallen, so you can use the appropriate *suvat* formula to find the acceleration of box Q.

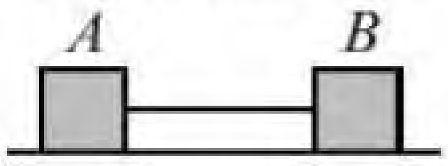
Problem solved!

In your AS exam, any resistances to the motion of an object (such as friction or air resistance) will be modelled as being **constant**. In real life, air resistance increases as the speed of an object increases. If you are asked to suggest an improvement to a model involving a constant resistance, you can suggest that the resistance is modelled as a **variable force** instead.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

A trailer A of mass m_1 kg is being pulled along a rough horizontal road by a truck B of mass m_2 kg. The truck and the trailer are connected by means of a tow rope, which is modelled as a taut, light inextensible string.



The total resistances to motion experienced by the trailer and the truck are modelled as being constant forces of magnitude 1000 N and 1800 N respectively. The truck generates a driving force of 4000 N which causes the truck and trailer to accelerate at 0.4 ms^{-2}.

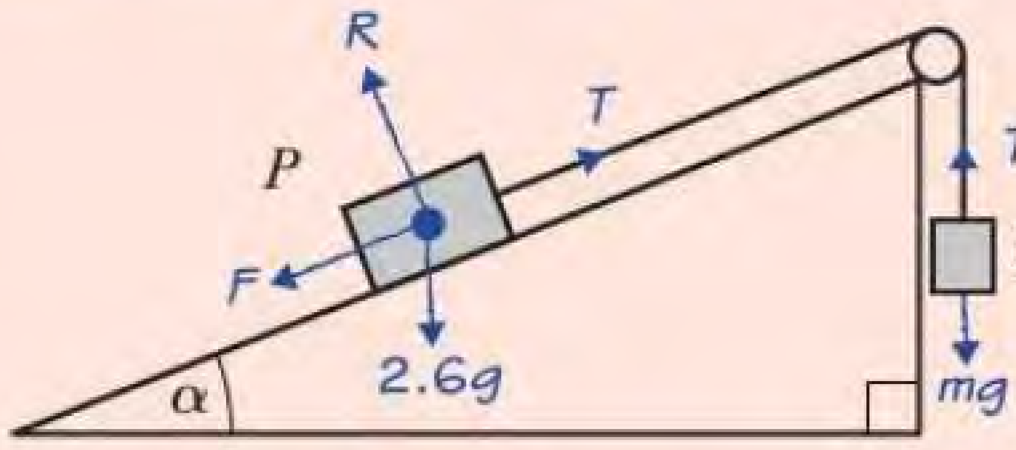
At the point when the truck and trailer are travelling at 14 ms^{-1}, the tow rope breaks, causing the trailer to come to rest in a time of 2.8 seconds.

Find the mass of the trailer and the mass of the truck. Answer **(9 marks)**

Connected particles 2

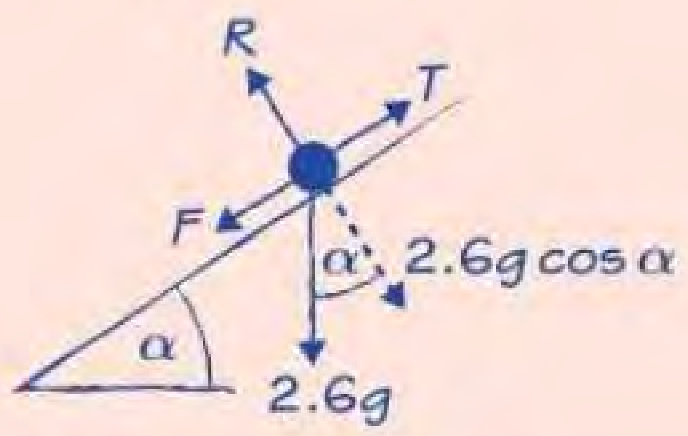
When two particles are connected via a pulley, one particle might be on a **plane**. If the plane is **rough**, you'll have to consider **friction** as well. Have a look at page 168 for a reminder.

Worked example



Two particles P and Q have masses 2.6 kg and m kg respectively. The particles are attached to the ends of a light inextensible string that passes over a smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$ The coefficient of friction between P and the plane is 0.5. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut. The system is released from rest and Q accelerates vertically downwards at $1.8\,\text{m s}^{-2}$. Find

(a) the magnitude of the normal reaction of the inclined plane on P **(2 marks)**



R(↖): $R - 2.6g\cos\alpha = 0$

Using $\cos\alpha = \frac{12}{13}$: $R = 2.6g \times \frac{12}{13}$

$= 24\,\text{N}$ (2 s.f.)

(b) the value of m. **(8 marks)**

R(↗): Using $F = ma$ for P to find T:

$T - F - 2.6g\sin\alpha = 2.6 \times 1.8$

Using $\sin\alpha = \frac{5}{13}$: $T - F = 14.48$ ①

$F = \mu R = 0.5 \times 24 = 12$

Substituting into ①: $T - 12 = 14.48$

$T = 26.48\,\text{N}$

R(↓): Using $F = ma$ on Q:

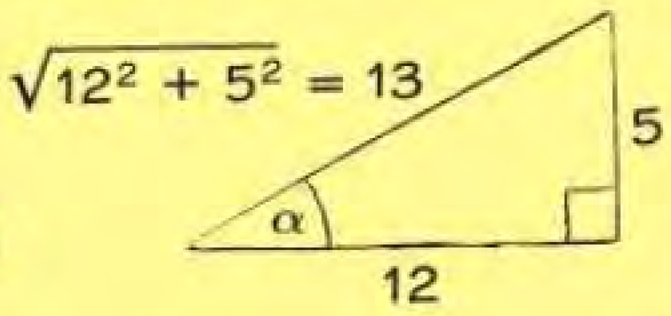
$mg - T = 1.8m$

$9.8m - 26.48 = 1.8m$

$8m = 26.48$

$m = 3.31 = 3.3\,\text{kg}$ (2 s.f.)

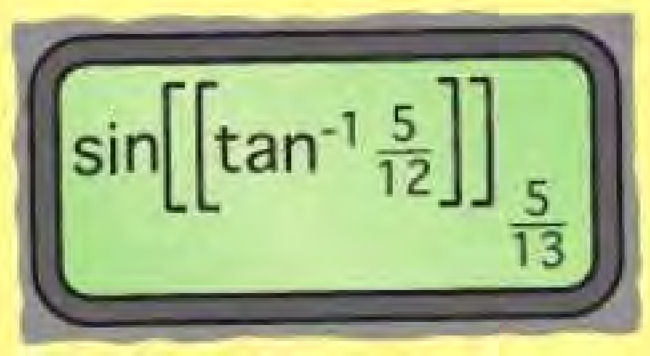
You have been given $\tan\alpha = \frac{5}{12}$. You can sketch a right-angled triangle to work out exact values of $\sin\alpha$ and $\cos\alpha$:



$\sin\alpha = \frac{5}{13}$

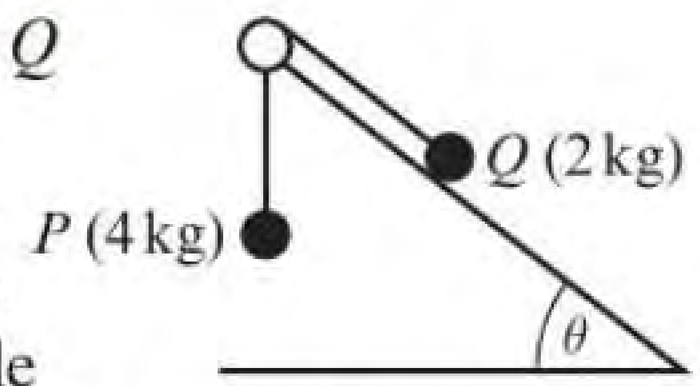
$\cos\alpha = \frac{12}{13}$

You might also be able to find these fractions on your calculator:



Resolve **parallel** to the plane to write an equation of motion for P, and resolve **vertically** to write an equation of motion for Q.

Now try this



Two particles P and Q of masses 4 kg and 2 kg respectively are attached to the ends of a light inextensible string, which runs over a smooth pulley. Q is held at rest on a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$. The coefficient of friction between Q and the plane is $\frac{1}{4}$. The particles are released from rest and Q accelerates up the plane.

(a) Find the acceleration of Q. **(10 marks)**

Answer

After 0.6 s, particle P hits the floor and remains there. Particle Q continues up the slope, reaching its highest point after a further T s. In this motion Q does not reach the pulley.

(b) Find the value of T. **(6 marks)**

Answer

You need to use the ***suvat*** formulae for part (b). u will be the speed of Q after 0.6 s, and v will be 0.

Have a look at pages 150 and 151 for a reminder.