Summary of key points

1 You can use the laws of indices to simplify powers of the **same base**.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

6 You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
- Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Index laws

You need to be able to work with algebraic expressions confidently for all of your AS maths topics. Make sure you know how to use these six index laws.

1. $a^m \times a^n = a^{m+n}$ $\quad x^5 \times x^{-2} = x^3$
2. $\frac{a^m}{a^n} = a^{m-n}$ $\quad \frac{x^8}{x^6} = x^2$
3. $(a^m)^n = a^{mn}$ $\quad (x^4)^3 = x^{12}$
4. $a^{-n} = \frac{1}{a^n}$ $\quad 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
5. $a^{\frac{1}{n}} = \sqrt[n]{a}$ $\quad 49^{\frac{1}{2}} = 7$, $27^{\frac{1}{3}} = 3$, $16^{\frac{1}{4}} = 2$

   $2 \times 2 \times 2 \times 2 = 16$ so $\sqrt[4]{16} = 2$
6. $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ $\quad 27^{-\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^{-2} = (\sqrt[3]{27})^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

   Do these calculations one step at a time.

Worked example

Given that $\frac{10x - 5x^{\frac{7}{2}}}{\sqrt{x}}$ can be written in the form $10x^p - 5x^q$, write down the value of p and the value of q. **(2 marks)**

$$\frac{10x}{x^{\frac{1}{2}}} - \frac{5x^{\frac{7}{2}}}{x^{\frac{1}{2}}} = 10x^{1-\frac{1}{2}} - 5x^{\frac{7}{2}-\frac{1}{2}}$$
$$= 10x^{\frac{1}{2}} - 5x^3$$
$p = \frac{1}{2}$ and $q = 3$

The fraction acts like a bracket, so you need to divide **both** terms in the numerator by $\sqrt{x}$. Write down the values of p and q when you've finished.

Golden rule

Convert all roots into fractional powers before applying the other index laws.

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad \sqrt[3]{x} = x^{\frac{1}{3}}$

You will be allowed to use your calculator in both of your AS exams, but be careful. If you enter $125^{-\frac{1}{3}}$ into your calculator it will give you the answer $\frac{1}{5}$ or 0.2. You need to be able to write it in the form given in the question.

Worked example

Express $125^{-\frac{1}{3}}$ in the form 5^n where n is an integer. **(1 mark)**

$125^{-\frac{1}{3}} = (5^3)^{-\frac{1}{3}}$
$= 5^{3\times(-\frac{1}{3})}$
$= 5^{-1}$
So $n = -1$

Check that your answer is in the correct form. n is an integer so it is a positive or negative whole number, or 0.

Now try this

1 Simplify $x(4x^{-\frac{1}{2}})^3$ Answer **(2 marks)**

2 Simplify $(9y^{10})^{\frac{3}{2}}$ Answer **(2 marks)**

3. Write $\frac{5 + 2\sqrt{x}}{x^2}$ in the form $5x^p + 2x^q$, where p and q are constants. **(2 marks)** Answer

Remember that a constant doesn't have to be an integer.

There are full worked solutions to all the Now try this questions on page 96.

Surds

You can simplify surds quickly using the fraction and square root functions on your calculator. However, you need to understand how to manipulate surds in order to work with algebraic expressions confidently.

Golden rules

These are the two golden rules for simplifying surds:

1. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

   $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\quad \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5}$

Worked example

Write $\sqrt{45}$ in the form $k\sqrt{5}$, where k is an integer. **(2 marks)**

$$\sqrt{45} = \sqrt{9 \times 5}$$
$$= \sqrt{9} \times \sqrt{5}$$
$$= 3\sqrt{5}$$
$$k = 3$$

Worked example

Simplify $\sqrt{27} + \sqrt{48}$, giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

$$\sqrt{27} + \sqrt{48} = \sqrt{9} \times \sqrt{3} + \sqrt{16} \times \sqrt{3}$$
$$= 3\sqrt{3} + 4\sqrt{3}$$
$$= 7\sqrt{3}$$
$$a = 3$$

$\sqrt{a} + \sqrt{b}$ is **not** equal to $\sqrt{a + b}$.

You know you need to write the answer in the form $a\sqrt{3}$ so write each surd in the form $k\sqrt{3}$. You need to take a factor of 3 out of each number:

$27 = 9 \times 3$ so $\sqrt{27} = \sqrt{9 \times 3}$

$48 = 16 \times 3$ so $\sqrt{48} = \sqrt{16 \times 3}$

Rationalising the denominator

You can rationalise the denominator of a fraction by removing any surds in the denominator.

$$\frac{1}{4 - \sqrt{11}} \xrightarrow{\times (4 + \sqrt{11})} \frac{4 + \sqrt{11}}{(4 - \sqrt{11})(4 + \sqrt{11})} = \frac{4 + \sqrt{11}}{5}$$

$$(4 - \sqrt{11})(4 + \sqrt{11}) = 16 - \cancel{4\sqrt{11}} + \cancel{4\sqrt{11}} - 11$$
$$= 5$$

To work out what to multiply the top and bottom by, look at the denominator of the original fraction. Swap a plus for a minus, or swap a minus for a plus.

Worked example

Express $\frac{14}{3 + \sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. **(2 marks)**

$$\frac{14}{3 + \sqrt{2}} = \frac{14(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$$
$$= \frac{14(3 - \sqrt{2})}{9 + 3\sqrt{2} - 3\sqrt{2} - 2}$$
$$= \frac{{}^{2}\cancel{14}(3 - \sqrt{2})}{\cancel{7}_{1}}$$
$$= 6 - 2\sqrt{2}$$
$$a = 6 \text{ and } b = -2$$

If the denominator is in the form $p + \sqrt{q}$ then multiply the numerator and denominator of the fraction by $p - \sqrt{q}$.

Now try this

1 Expand and simplify $(x + \sqrt{3})(x - \sqrt{3})$. Answer **(2 marks)**

2 Write $\sqrt{98}$ in the form $a\sqrt{2}$, where a is an integer. Answer **(1 mark)**

3 Simplify the following, giving your answers in the form $a + b\sqrt{5}$, where a and b are integers.

(a) $\frac{8}{3 + \sqrt{5}}$ **(2 marks)** Answer (b) $\frac{4 + \sqrt{5}}{2 - \sqrt{5}}$ **(4 marks)** Answer