Summary of key points

- You can use the laws of indices to simplify powers of the same base.
 - $a^m \times a^n = a^{m+n}$
 - $(a^m)^n = a^{mn}$
- Factorising is the opposite of expanding brackets.
- A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$. 3

$$4 \quad x^2 - y^2 = (x + y)(x - y)$$

5 You can use the laws of indices with any rational power.

•
$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

• $a^{-m} = \frac{1}{a^m}$
• $a^0 = 1$

- You can manipulate surds using these rules:
 - $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ • $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- The rules to rationalise denominators are:
 - Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by \sqrt{a} .
 - Fractions in the form $\frac{1}{a+\sqrt{b}}$, multiply the numerator and denominator by $a \sqrt{b}$.
 - Fractions in the form $\frac{1}{a-\sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

- $a^m \div a^n = a^{m-n}$
- $(ab)^n = a^n b^n$



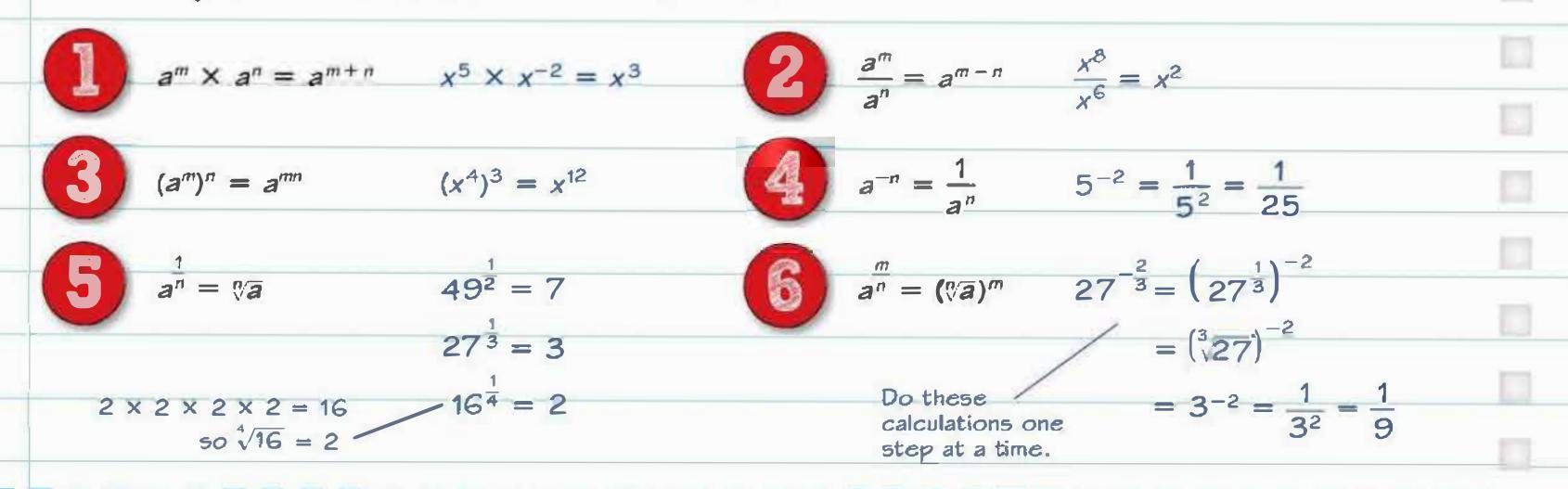
Index laws

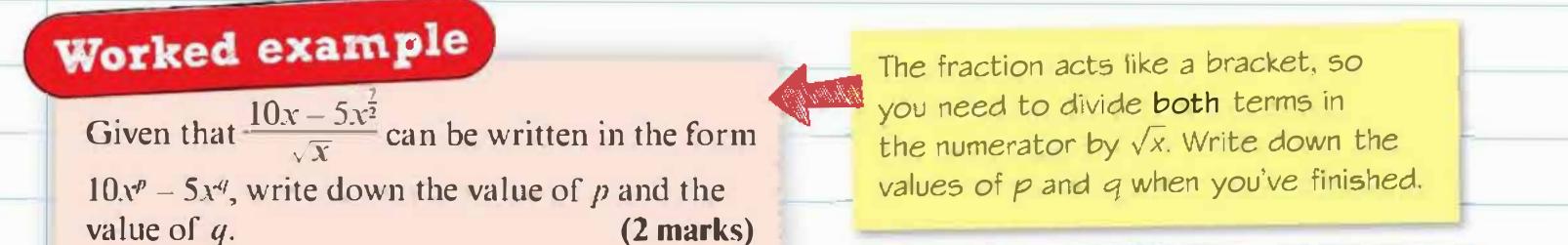
Nearly there

Nailed it!

Had a look

You need to be able to work with algebraic expressions confidently for all of your AS maths topics. Make sure you know how to use these six index laws.





$$\frac{10x}{x^2} - \frac{5x^2}{x^2} = 10x^{1-\frac{1}{2}} - 5x^{\frac{7}{2}-\frac{1}{2}}$$
$$= 10x^{\frac{1}{2}} - 5x^3$$
$$p = \frac{1}{2} \text{ and } q = 3$$

Golden rule Convert all roots into fractional powers before applying the other index laws. $\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x} = x^{\frac{1}{3}}$

You will be allowed to use your calculator in both of your AS exams, but be careful. If you enter 125 into your calculator it will give you the answer $\frac{1}{5}$ or 0.2. You need to be able to write it in the form given in the question.

Check that your answer is in the correct form *n* is an integer so it is a positive or negative whole number, or O.

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Worked example

Express $125^{-\frac{1}{3}}$ in the form 5" where *n* is an integer. (1 mark) $125^{-\frac{1}{3}} = (5^3)^{-\frac{1}{3}}$ $= 5^{3 \times (-\frac{1}{3})}$ $= 5^{-1}$ So n = -1

Now try this1 Simplify $x(4x^{-\frac{1}{2}})^3$ (2 marks)2 Simplify $(9y^{10})^{\frac{3}{2}}$ (2 marks)3. Write $\frac{5+2\sqrt{x}}{x^2}$ in the form $5x^p + 2x^q$, wherep and q are constants.(2 marks)There are full worked solutions to all the
Now try this questions on page 96.Remember that a constant
doesn't have to be an integer.

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Nailed it!



You can simplify surds quickly using the fraction and square root functions on your calculator. However, you need to understand how to manipulate surds in order to work with algebraic expressions confidently.

Golden rules	Worked example
These are the two golden rules for simplifying surds: $ \begin{array}{c} 1 \\ \sqrt{ab} = \sqrt{a} \times \sqrt{b} \\ \sqrt{a} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2} \\ \end{array} $ $ \begin{array}{c} 2 \\ \sqrt{a} \\ \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} \\ \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5} \end{array} $	Write $\sqrt{45}$ in the form $k\sqrt{5}$, where k is an integer. (2 marks) $\sqrt{45} = \sqrt{9 \times 5}$ $= \sqrt{9} \times \sqrt{5}$ $= 3\sqrt{5}$ k = 3
	$\sqrt{a} + \sqrt{b}$ is not equal to $\sqrt{a} + b$.
Simplify $\sqrt{27} + \sqrt{48}$, giving your answer in the form $a\sqrt{3}$, where <i>a</i> is an integer. (2 marks) $\sqrt{27} + \sqrt{48} = \sqrt{9} \times \sqrt{3} + \sqrt{16} \times \sqrt{3}$	You know you need to write the answer in the form $a\sqrt{3}$ so write each surd in the form $k\sqrt{3}$. You need to take a factor of 3 out of each

$$= 3\sqrt{3} + 4\sqrt{3}$$
$$= 7\sqrt{3}$$
$$= 3$$

$$27 = 9 \times 3 \text{ so } \sqrt{27} = \sqrt{9 \times 3}$$
$$48 = 16 \times 3 \text{ so } \sqrt{48} = \sqrt{16 \times 3}$$

Rationalising the denominator

You can rationalise the denominator of a fraction by removing any surds in the denominator.

$$\frac{\times (4 + \sqrt{11})}{1} = \frac{4 + \sqrt{11}}{(4 - \sqrt{11})(4 + \sqrt{11})} = \frac{4 + \sqrt{11}}{5}$$
$$\times (4 + \sqrt{11})$$
$$(4 - \sqrt{11})(4 + \sqrt{11}) = 16 - 4\sqrt{11} + 4\sqrt{11} - 11$$

To work out what to multiply the top and bottom by, look at the denominator of the original fraction. Swap a plus for a minus, or swap a minus for a plus.

Now try this

1 Expand and simplify $(x + \sqrt{3})(x - \sqrt{3})$.

(2 marks)

2 Write $\sqrt{98}$ in the form $a\sqrt{2}$, where *a* is an integer. (1 mark)

Worked example

Express
$$\frac{14}{3 + \sqrt{2}}$$
 in the form $a + b\sqrt{2}$,
where *a* and *b* are integers. (2 marks)
 $\frac{14}{3 + \sqrt{2}} = \frac{14(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$
 $= \frac{14(3 - \sqrt{2})}{9 + 3\sqrt{2} - 3\sqrt{2} - 2}$
 $= \frac{214(3 - \sqrt{2})}{\sqrt{1}}$
 $= 6 - 2\sqrt{2}$
 $a = 6$ and $b = -2$

If the denominator is in the form $p + \sqrt{q}$ then multiply the numerator and denominator of the fraction by $p - \sqrt{q}$.

- 3 Simplify the following, giving your answers in the form $a + b\sqrt{5}$, where a and b are integers.
 - (a) $\frac{8}{3+\sqrt{5}}$ (2 marks) (b) $\frac{4+\sqrt{5}}{2-\sqrt{5}}$ (4 marks)

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