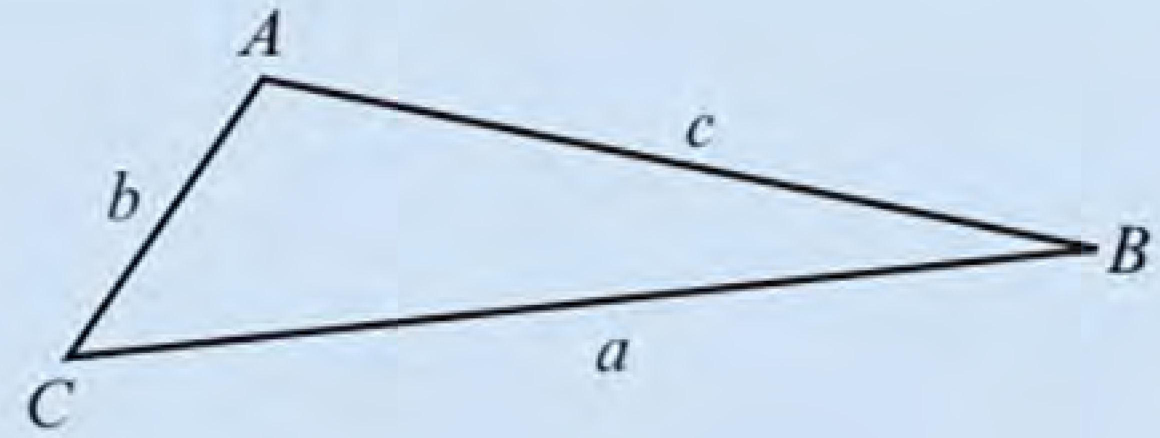
Summary of key points

1 This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc \cos A$$



2 This version of the cosine rule is used to find an angle if you know all three sides:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 This version of the sine rule is used to find the length of a missing side:

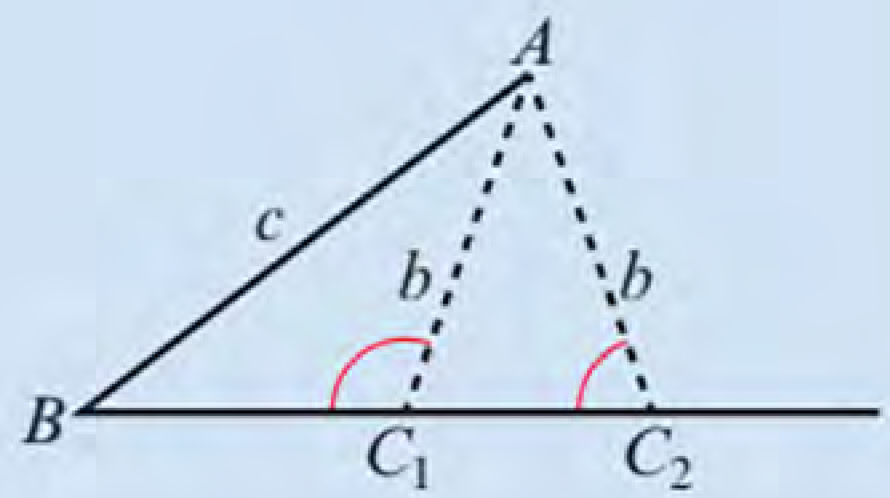
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4 This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

5 The sine rule sometimes produces two possible solutions for a missing angle:

$$\sin \theta = \sin (180° - \theta)$$



6 Area of a triangle $= \frac{1}{2}ab \sin C$.

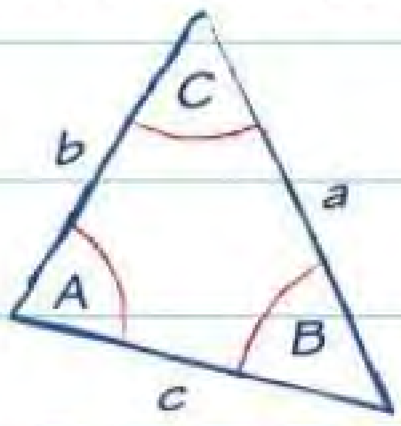
7 The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

- The graph of $y = \sin \theta$: repeats every 360° and crosses the x-axis at …, −180°, 0, 180°, 360°, …
  has a maximum value of 1 and a minimum value of −1.
- The graph of $y = \cos \theta$: repeats every 360° and crosses the x-axis at …, −90°, 90°, 270°, 450°, …
  has a maximum value of 1 and a minimum value of −1
- The graph of $y = \tan \theta$: repeats every 180° and crosses the x-axis at … −180°, 0°, 180°, 360°, …
  has no maximum or minimum value
  has vertical asymptotes at $x = -90°$, $x = 90°$, $x = 270°$, …

Had a look ☐ Nearly there ☐ Nailed it! ☐

Cosine rule

The cosine rule applies to **any triangle.** You usually use the cosine rule when you know **two sides** and the **angle between them** (SAS) or when you are given **three sides** and you want to work out an **angle** (SSS).



1 $a^2 = b^2 + c^2 - 2bc\cos A$

Use this version to find a missing side.

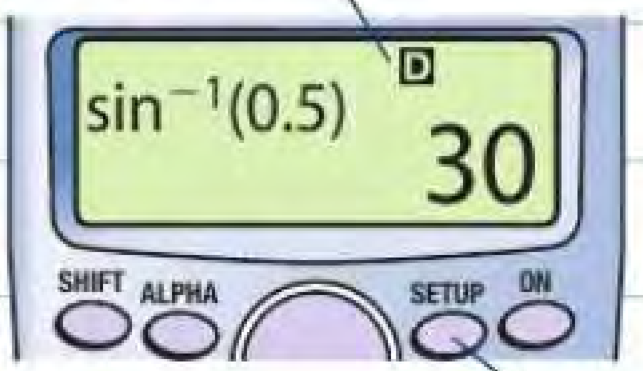
2 $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

This version is useful for finding a missing angle.

Using a calculator

Angles can be measured in degrees or radians. You don't need to know about radians in AS maths, but your calculator has a mode for them. Make sure your calculator is set in **degrees** mode for your AS exam.

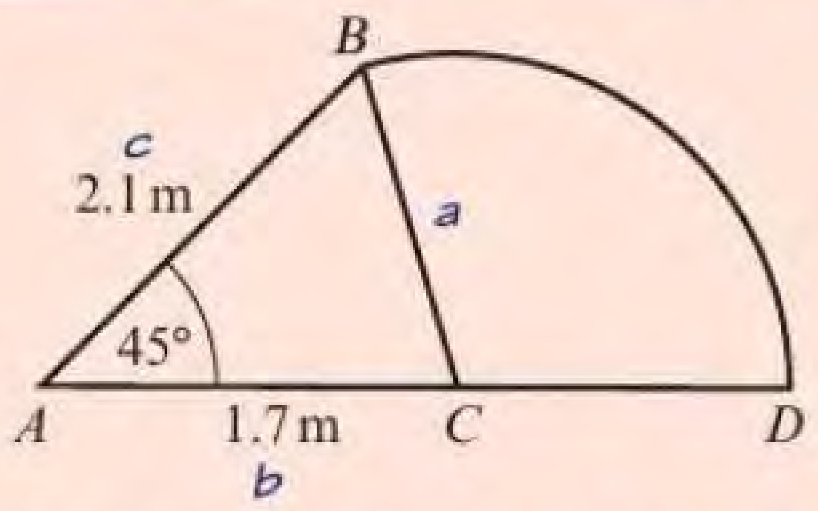
This calculator is in degrees mode.



You might need to use the **shift** key to enter inverse trigonometric functions like $\sin^{-1}$.

On some calculators you need to press SHIFT and SETUP to change between degrees and radians modes.

Worked example



Everything in blue is part of the answer.

The diagram shows a triangle ABC and a sector BCD of a circle with centre C.
Find the length of BC. **(3 marks)**

$a^2 = b^2 + c^2 - 2bc\cos A$

$BC^2 = 1.7^2 + 2.1^2 - 2 \times 1.7 \times 2.1 \times \cos 45°$

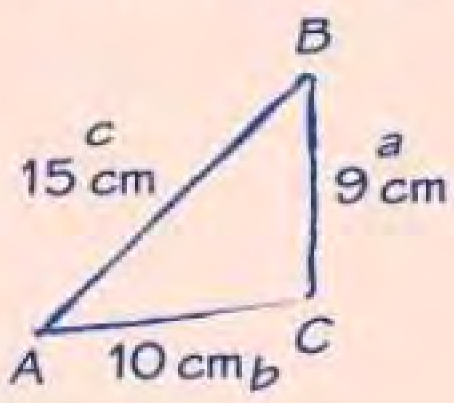
$= 2.2512...$

$BC = 1.50$ m (3 s.f.)

You know two sides and the angle between them (SAS) so you can use the cosine rule to find the opposite side. Label the sides with the lower case letter of the **opposite angle** and write out the formula before you substitute.

Worked example

In the triangle ABC, $AB = 15$ cm, $BC = 9$ cm and $CA = 10$ cm. Find the size of angle C, giving your answer to the nearest degree. **(3 marks)**



$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{9^2 + 10^2 - 15^2}{2 \times 9 \times 10} = -0.2444...$

$C = \cos^{-1}(-0.2444...) = 104°$

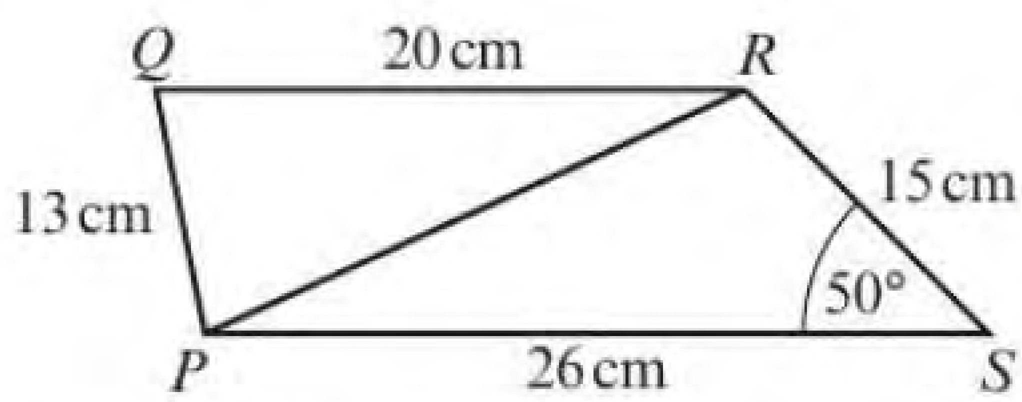
If no diagram is given in the question you can sketch one. You know three sides (SSS) so you can use the cosine rule to find any angle in the triangle. Be careful with the order. You add the squares of the sides **adjacent** to the angle, and subtract the square of the **opposite** side.

Now try this

The diagram shows two triangles PQR and PRS.
$\angle RSP = 50°$. Find

(a) the length of PR Answer **(3 marks)**

(b) the size of $\angle PQR$, giving your answer in degrees to 1 decimal place. Answer **(3 marks)**



Sine rule

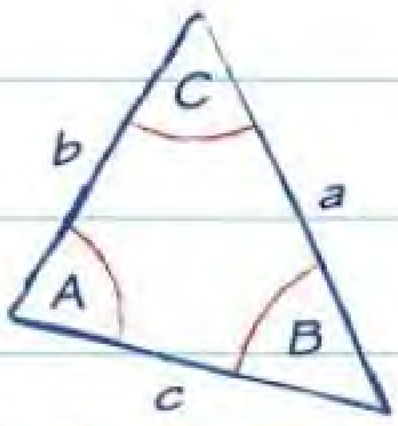
You need to **learn** the sine rule for your exam. It applies to **any triangle**. The sine rule is useful when you know **two angles**, or when you know a side and the **opposite** angle.

1 $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

This version is useful for finding a missing side.

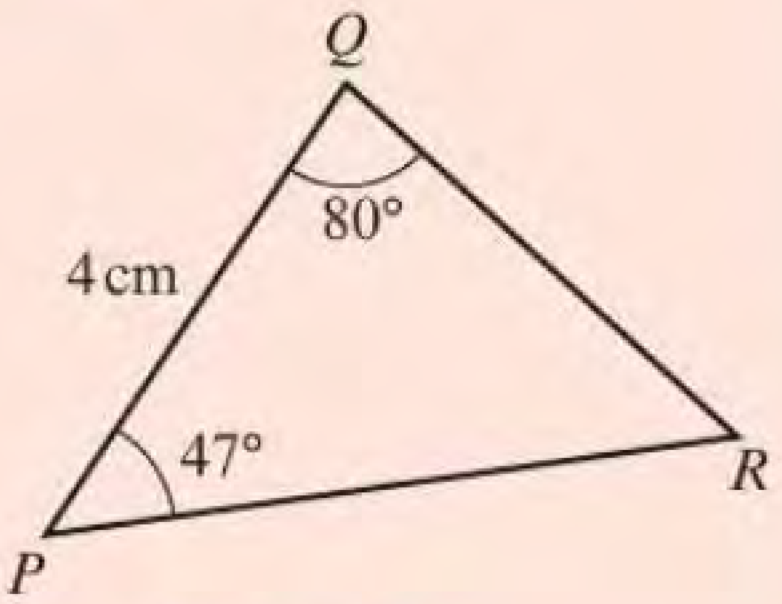
2 $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Use this version to find a missing angle.



Worked example

In the triangle PQR, $PQ = 4$ cm, $\angle PQR = 80°$ and $\angle QPR = 47°$. Find the length of the side PR and the area of the triangle. **(5 marks)**



$\angle QRP = 180° - 80° - 47° = 53°$

$\dfrac{PR}{\sin 80°} = \dfrac{4}{\sin 53°}$

$PR = \dfrac{4 \times \sin 80°}{\sin 53°} = 4.93$ cm (3 s.f.)

$\text{Area} = \dfrac{1}{2}ab\sin\theta$

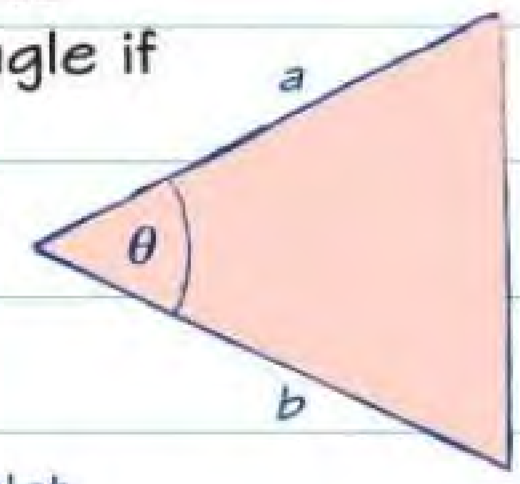
$= \dfrac{1}{2} \times 4 \times 4.93... \times \sin 47° = 7.21$ (3 s.f.)

The sine rule uses **opposite** sides and angles, so use the fact that the angles in a triangle add up to 180° to work out $\angle QRP$ first.

Areas of triangles

You can use this formula to find the area of **any** triangle if you know two sides and the angle between them (SAS):

$\text{Area} = \dfrac{1}{2}ab\sin\theta$



This formula is not in the booklet so you need to **learn** it.

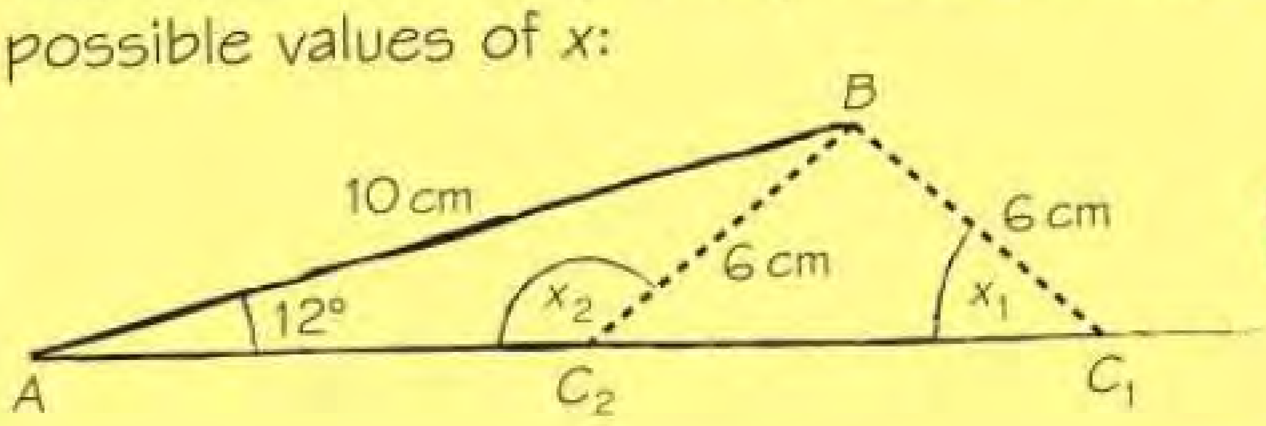
Two values

If you know $\sin x$, you might be asked to find **two possible values** for x. Use this rule:

$\sin x = \sin(180° - x)$

There is more about this on page 30.

This sketch shows you why there are two possible values of x:



Worked example

In the triangle ABC, $AB = 10$ cm, $BC = 6$ cm, $\angle BAC = 12°$ and $\angle ACB = x$.

(a) Find the value of $\sin x$, giving your answer to 3 decimal places. **(3 marks)**

$\dfrac{\sin x}{10} = \dfrac{\sin 12°}{6}$

$\sin x = \dfrac{10 \times \sin 12°}{6} = 0.347$ (3 d.p.)

(b) Given that there are two possible values of x, find these values of x, correct to 1 decimal place. **(3 marks)**

$x_1 = \sin^{-1}(0.3465...) = 20.3°$ (1 d.p.)

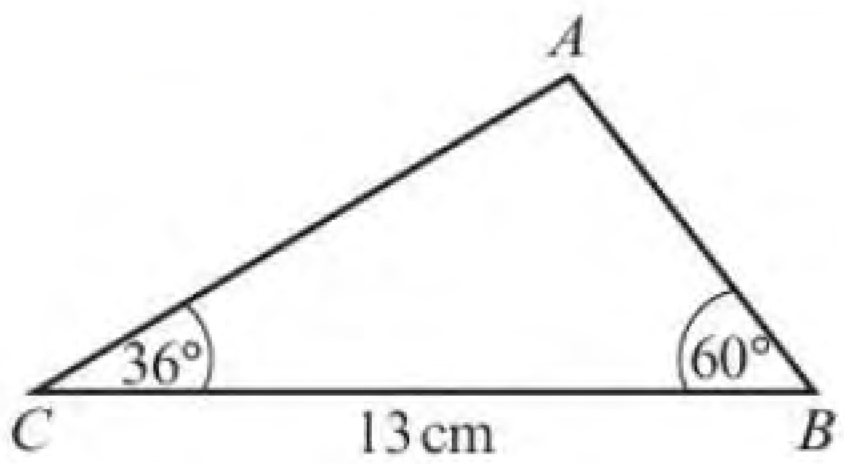
$x_2 = 180° - x_1 = 180° - 20.3° = 159.7°$ (1 d.p.)

Now try this

(a) In the triangle ABC, $BC = 13$ cm, $\angle ABC = 60°$, and $\angle ACB = 36°$. Find the length of AC. Answer **(3 marks)**

(b) Find the area of the triangle. Answer **(2 marks)**

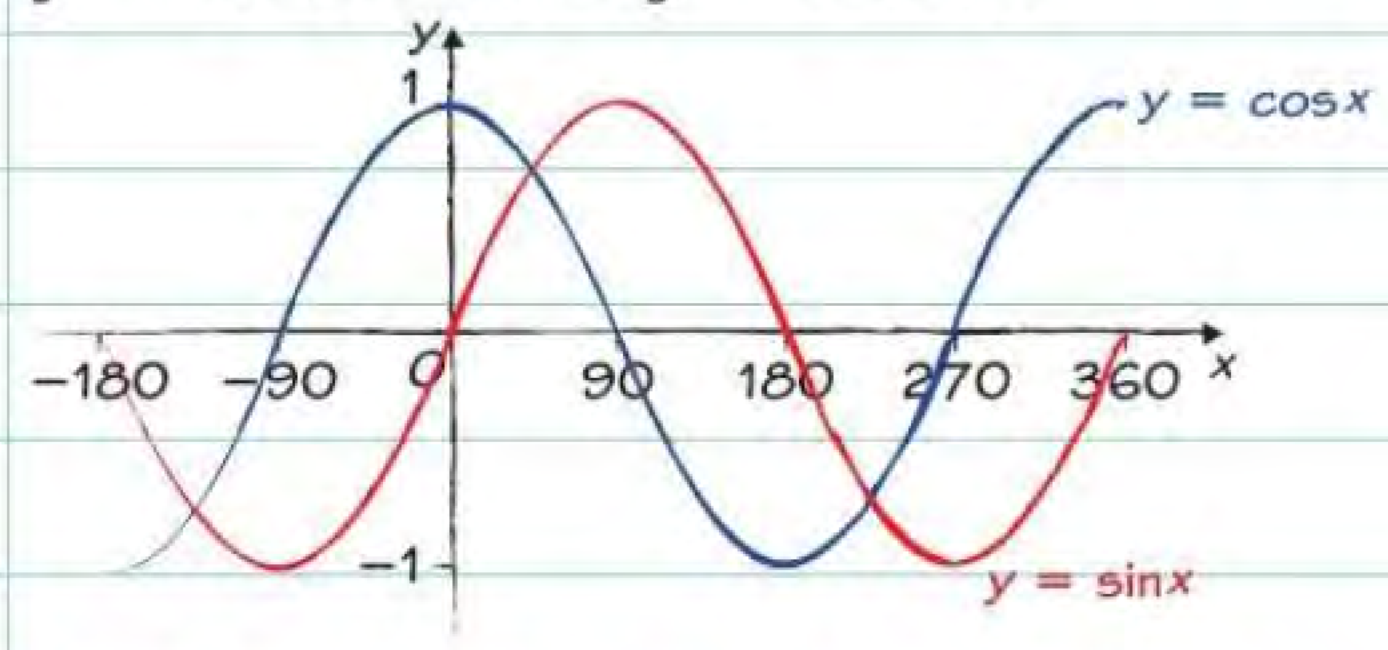
Start by finding the size of $\angle CAB$.



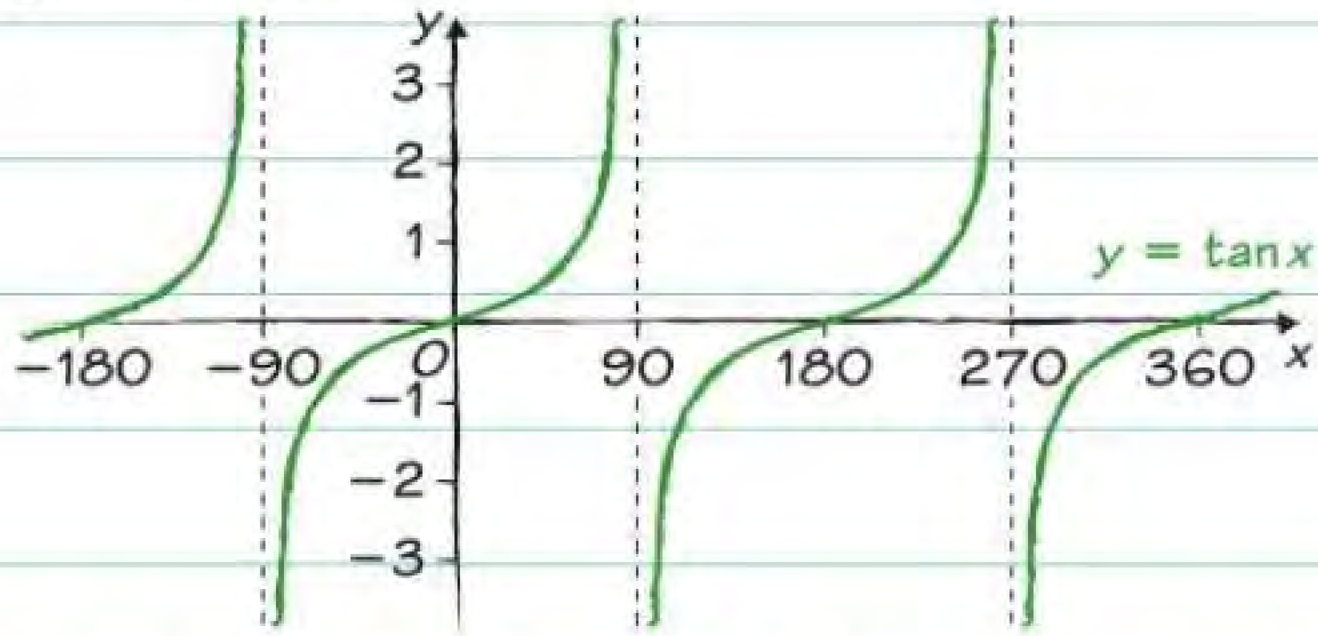
Trigonometric graphs

You need to be able to sketch the graphs of **sin**, **cos** and **tan**, and **transformations** of them. If you want to recap transformations of graphs, have a look at pages 13 and 14.

$y = \sin x$ and $y = \cos x$



$y = \tan x$



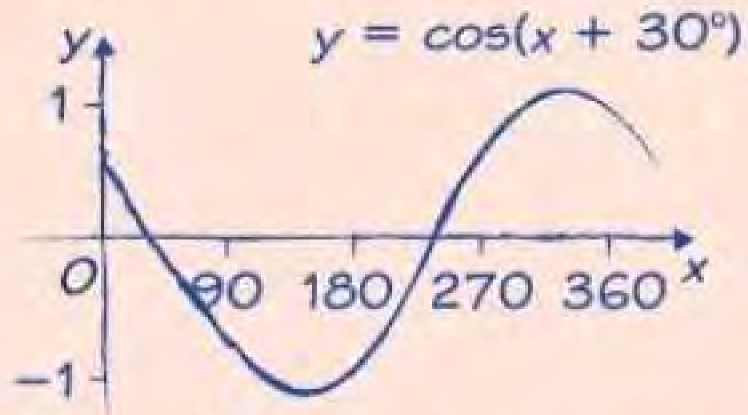
Sketching trig graphs

If you have to sketch a trigonometric graph in your exam, make sure you:

- pay attention to the **range** of values for x in the question
- label multiples of 90° on the x-axis
- put a scale on the y-axis to show the **max** and **min** for $\sin x$ and $\cos x$
- draw the **asymptotes** for $\tan x$.

Worked example

(a) Sketch, for $0 \leqslant x \leqslant 360°$, the graph of $y = \cos(x + 30°)$ **(2 marks)**



(b) Write down the exact coordinates of the points where the graph meets the coordinate axes. **(3 marks)**

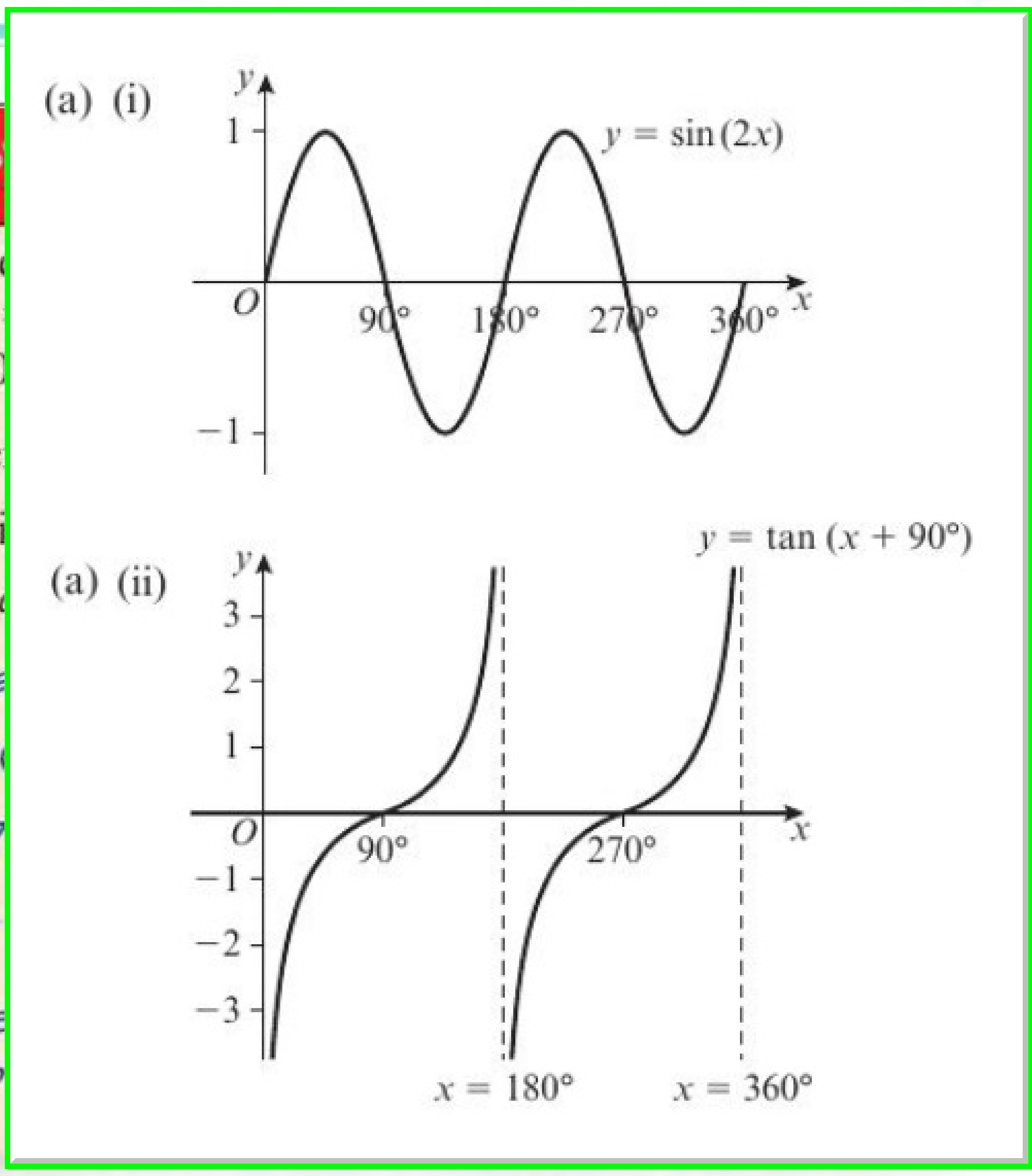
When $x = 0$, $y = \cos 30° = \frac{\sqrt{3}}{2}$, so $\left(0, \frac{\sqrt{3}}{2}\right)$

When $y = 0$, $0 = \cos(x + 30°)$:

$90° - 30° = 60°$, so $(60°, 0)$

and $270° - 30° = 240°$, so $(240°, 0)$

You can write cos 30° exactly as a surd. The graph of $y = (\cos x + 30°)$ is a **translation** of $y = \cos x$. The graph moves 30° to the **left**.



Wor

The
of y
a > 0

Give
x-axi
find

sin (a

a(21

a(57

② –

Subs
so b

If $\sin(ax - b) = 0$, then $ax - b = 0$, or 180°, or 360° and so on. You can use these facts to write two equations and solve them **simultaneously** to find a and b.

Now try this

(a) On separate diagrams, sketch, for $0 \leqslant x \leqslant 360°$, the graphs of

(i) $y = \sin(2x)$

(ii) $y = \tan(x + 90°)$ **(4 marks)** Answer

(b) Write down the coordinates of any points where the curves meet the coordinate axes and the equations of any asymptotes. **(6 marks)** Answer