

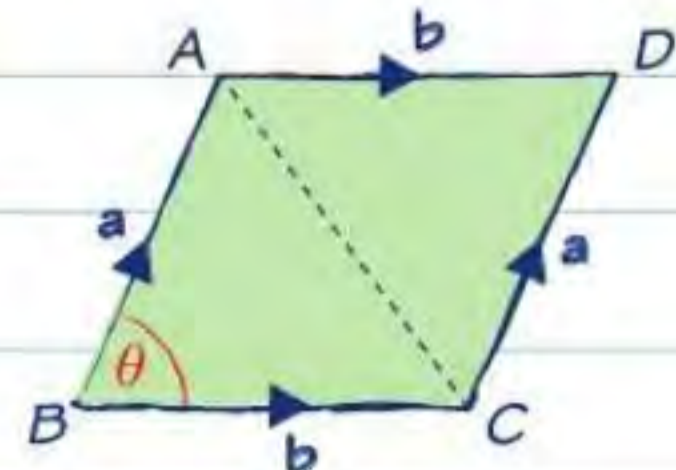
Solving vector problems

You can use these formulae to find areas of triangles and parallelograms in vector questions:

1 Area of triangle $ABC = \frac{1}{2} |a| |b| \sin \theta$

2 Area of parallelogram $ABCD = |a| |b| \sin \theta$

The area of the parallelogram is twice the area of the triangle.



Worked example

Three points are such that $\vec{AB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\vec{AC} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$. Find the area of the triangle ABC . (6 marks)

$$\begin{aligned} \vec{CB} &= \vec{CA} + \vec{AB} \\ &= -\vec{AC} + \vec{AB} \\ &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \end{aligned}$$

$$AB = \sqrt{3^2 + 6^2} = \sqrt{45}$$

$$AC = \sqrt{4^2 + 1^2} = \sqrt{17}$$

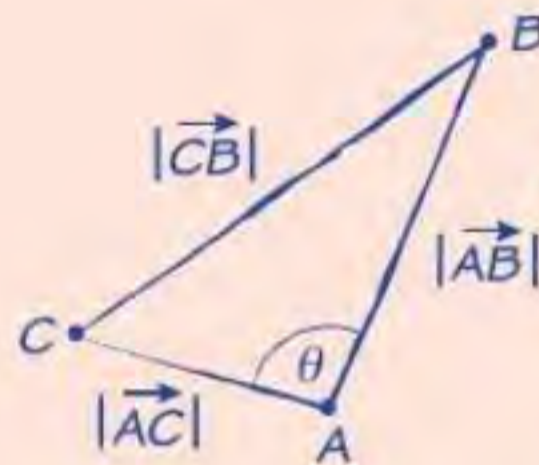
$$CB = \sqrt{7^2 + 5^2} = \sqrt{74}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \theta = \frac{45 + 17 - 74}{2\sqrt{45}\sqrt{17}} = -\frac{2}{\sqrt{85}}$$

$$\theta = 102.528\dots^\circ$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin \theta = \frac{1}{2} \sqrt{45} \sqrt{17} \sin 102.528\dots^\circ \\ &= 13.5 \end{aligned}$$



Problem solved!

Draw a sketch to help you visualise the problem. This will help you plan your strategy before you start.

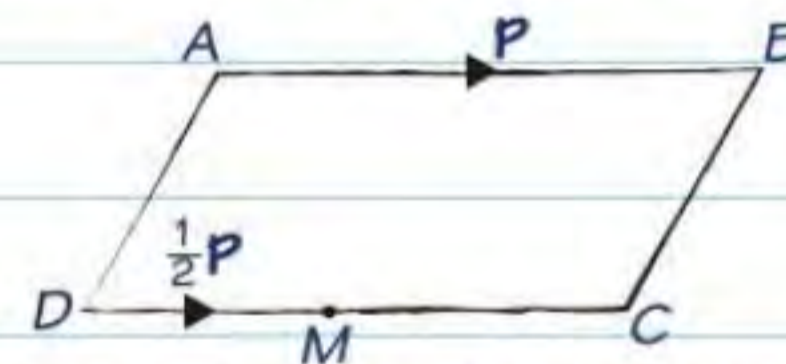
1. Find \vec{CB} (or \vec{BC}) then find the lengths of all three sides of the triangle.
2. Use the **cosine rule** to find the size of one angle.
3. Use $\text{Area} = \frac{1}{2} ab \sin \theta$ to find the area of the triangle.

You will need to use problem-solving skills throughout your exam – **be prepared!**



Parallel vectors

If one vector can be written as a multiple of the other then the vectors are **parallel**.



In this parallelogram M is the midpoint of DC . AB is parallel to DM so $\vec{DM} = \frac{1}{2} \vec{AB}$

Now try this

1 Points A , B and C have position vectors

$$\begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ 5 \end{pmatrix} \text{ respectively.}$$

(a) Find the vectors \vec{AB} , \vec{BC} and \vec{CA} . (3 marks)

(b) Find the area of triangle ABC . (4 marks)

Point D is such that the points A , B , C and D form the vertices of a parallelogram.

(c) Write down the area of the parallelogram. (1 mark)

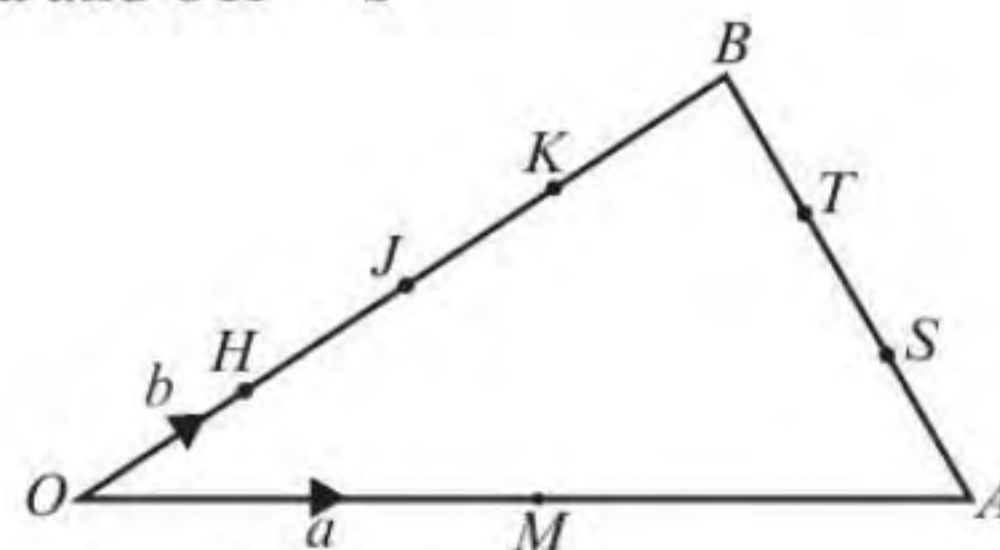
(d) Find the position vector of three possible positions of D . (4 marks)

2 In triangle OAB , M is the midpoint of OA .

$$OH = HJ = JK = KB$$

S and T divide the line AB into three equal segments.

$$\vec{OM} = \mathbf{a} \text{ and } \vec{OH} = \mathbf{b}$$



(a) Prove that \vec{HA} , \vec{JS} and \vec{KT} are all parallel. (6 marks)

(b) State the ratio $KT : JS : HA$ (1 mark)