Summary of key points

12

- $\frac{d}{dx}(f(y)) = f'(y)\frac{dy}{dx}$
- $\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$
- $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$

Implicit differentiation

An **implicit** equation is one that cannot easily be written in the form $y = f(x)$ or $x = g(y)$.
You can differentiate an implicit equation to find an expression for $\frac{dy}{dx}$ in terms of x and y. Follow these steps:

- Differentiate **every term** on **both sides** of the equation **with respect to** x.
- Collect terms involving $\frac{dy}{dx}$ on one side and the remaining terms on the other side.
- Factorise to get $\frac{dy}{dx}$ on its own.

Golden rules

These rules will help you with implicit differentiation in your exam:

1. $\frac{d}{dx}[f(y)] = f'(y)\frac{dy}{dx}$

   $\frac{d}{dx}[e^{2y}] = 2e^{2y}\frac{dy}{dx}$

2. $\frac{d}{dx}[g(x)y] = g'(x)y + g(x)\frac{dy}{dx}$

   $\frac{d}{dx}[x^3y] = 3x^2y + x^3\frac{dy}{dx}$

Problem solved!

Make sure you differentiate the **constant term** on the right-hand side:

$$\frac{d}{dx}[1] = 0$$

You can use golden rule 1 above to differentiate $3\cos 2y$ with respect to x. Remember that you have x **and** y in your expression for $\frac{dy}{dx}$ so you probably won't be able to simplify it very much.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Worked example

A set of curves satisfy the equation

$$6\sin 2x + 3\cos 2y = 1$$

Find $\frac{dy}{dx}$ in terms of x and y. **(3 marks)**

$$12\cos 2x - 6\sin 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2\cos 2x}{\sin 2y}$$

Worked example

A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0$$

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(7 marks)**

$$6x + 8y\frac{dy}{dx} - 2 + 6y + 6x\frac{dy}{dx} = 0$$

$$8y\frac{dy}{dx} + 6x\frac{dy}{dx} = 2 - 6y - 6x$$

$$\frac{dy}{dx}(8y + 6x) = 2 - 6y - 6x$$

$$\frac{dy}{dx} = \frac{2 - 6y - 6x}{8y + 6x} = \frac{1 - 3y - 3x}{4y + 3x}$$

At the point $(1, -2)$, $\frac{dy}{dx} = \frac{1 - 3(-2) - 3(1)}{4(-2) + 3(1)} = -\frac{4}{5}$

Equation of tangent: $y - (-2) = -\frac{4}{5}(x - 1)$

$$4x + 5y + 6 = 0$$

Be careful with the $6xy$ term. You can use the product rule (golden rule 2 above). Once you have differentiated every term, you need to rearrange the equation so all the $\frac{dy}{dx}$ terms are on one side. You can then factorise to get $\frac{dy}{dx}$ on its own, then divide by the other factor. To find the gradient at $(1, -2)$ you need to substitute $x = 1$ and $y = -2$ into your expression for $\frac{dy}{dx}$.

Now try this

1 The point P with coordinates $(3, -1)$ lies on the curve with equation
$x^3 + y^2 + 3x^2y = 1$
Show that at P, $\frac{dy}{dx} = -\frac{9}{25}$ **(5 marks)**

Answer

2 The curve C is described by the equation
$2x^2 - y^2 = ye^{3x}$

(a) Show that the point $(0, -1)$ lies on C. **(1 mark)**

Answer

(b) Find an equation of the tangent to C at the point $(0, -1)$. **(7 marks)**

Answer

$$\frac{d}{dx}(ye^{3x}) = 3ye^{3x} + e^{3x}\frac{dy}{dx}$$