Summary of key points

7 The **integration by parts** formula is given by: $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Integration by parts

You can integrate some functions written as a **product** of two functions using integration by parts. This rule is given in the formulae booklet:

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$u\frac{dv}{dx}$ is the function you want to integrate. You have to work out which part of the function to set as u and which part to set as $\frac{dv}{dx}$.

Always write down which part of the function you are setting as u and which part you are setting as $\frac{dv}{dx}$. Then calculate $\frac{du}{dx}$ and v and write them down before substituting. You don't need to include a constant of integration when you work out v but you should include one at the end of your final integral.

Choosing u and $\frac{dv}{dx}$

Follow these rules for choosing which parts of the function to set as u and $\frac{dv}{dx}$ in your exam:

1. **always** set $\ln x$ as u
2. If there's no $\ln x$, set x or x^2 as u
3. Set e^x, $\sin x$ or $\cos x$ as $\frac{dv}{dx}$

Worked example

(a) Use integration by parts to find $\int x \sin 3x\, dx$ **(3 marks)**

$u = x \qquad \frac{dv}{dx} = \sin 3x$

$\frac{du}{dx} = 1 \qquad v = -\frac{1}{3}\cos 3x$

$\int x \sin 3x\, dx = (x)\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)(1)\, dx$

$= -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\, dx$

$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$

(b) Using your answer to part (a), find $\int x^2 \cos 3x\, dx$ **(3 marks)**

$u = x^2 \qquad \frac{dv}{dx} = \cos 3x$

$\frac{du}{dx} = 2x \qquad v = \frac{1}{3}\sin 3x$

$\int x^2 \cos 3x\, dx = (x^2)\left(\frac{1}{3}\sin 3x\right) - \int\left(\frac{1}{3}\sin 3x\right)(2x)\, dx$

$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\int x\sin 3x\, dx$

$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) + c$

$= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + c$

Problem solved!

You can integrate $\ln x$ by writing it as $\int(\ln x)(1)\, dx$

You **always** set $u = \ln x$, so $\frac{du}{dx} = \frac{1}{x}$.

This means that the second part of the formula becomes:

$\int (x)\left(\frac{1}{x}\right) dx = \int 1\, dx$

You will need to use problem-solving skills throughout your exam – **be prepared!**

Worked example

Find the exact value of $\int_1^3 \ln x\, dx$ **(4 marks)**

$u = \ln x \qquad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{1}{x} \qquad v = x$

$\int_1^3 \ln x\, dx = \left[x\ln x\right]_1^3 - \int_1^3 1\, dx$

$= [x\ln x - x]_1^3$

$= (3\ln 3 - 3) - (1\ln 1 - 1)$

$= 3\ln 3 - 2$

Now try this

1 Use integration by parts to find $\int \frac{1}{x^2}\ln x\, dx$ Answer **(4 marks)**

2 (a) Find $\int xe^x\, dx$ Answer **(3 marks)**

(b) Hence show that $\int_0^1 x^2e^x\, dx = e - 2$ Answer **(4 marks)**

You always set e^x as $\frac{dv}{dx}$