Summary of key points

10 The **trapezium rule** is:

$$\int_a^b y\,dx \approx \frac{1}{2}h(y_0 + 2(y_1 + y_2 \ldots + y_{n-1}) + y_n)$$

where $h = \dfrac{b-a}{n}$ and $y_i = f(a + ih)$.

The trapezium rule

There are some definite integrals that are difficult or even impossible to evaluate using calculus. You can use a **numerical method** called the trapezium rule to find an **approximation**.

The formulae booklet contains this formula for using the trapezium rule with ***n* strips**.

Numerical Methods

The trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

y_0, y_1, y_2, … and so on are values of y which you will calculate in a table of values. There are $n + 1$ values for y in total.

Worked example

(a) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$. **(2 marks)**

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1	1.01959	1.08239	1.20269	1.41421

(b) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x\,dx$. Show all the steps of your working and give your answer to 4 decimal places. **(3 marks)**

$n = 4$, $a = 0$, $b = \frac{\pi}{4}$, $h = \frac{\pi}{16}$

$$\int_0^{\frac{\pi}{4}} \sec x\,dx \approx \frac{1}{2} \times \frac{\pi}{16}\left[(1 + 1.41421) + 2(1.01959 + 1.08239 + 1.20269)\right]$$

$$= 0.8859 \text{ (4 d.p.)}$$

The exact value of $\int_0^{\frac{\pi}{4}} \sec x\,dx$ is $\ln(1 + \sqrt{2})$.

(c) Calculate the % error in using the estimate you obtained in part (b). **(2 marks)**

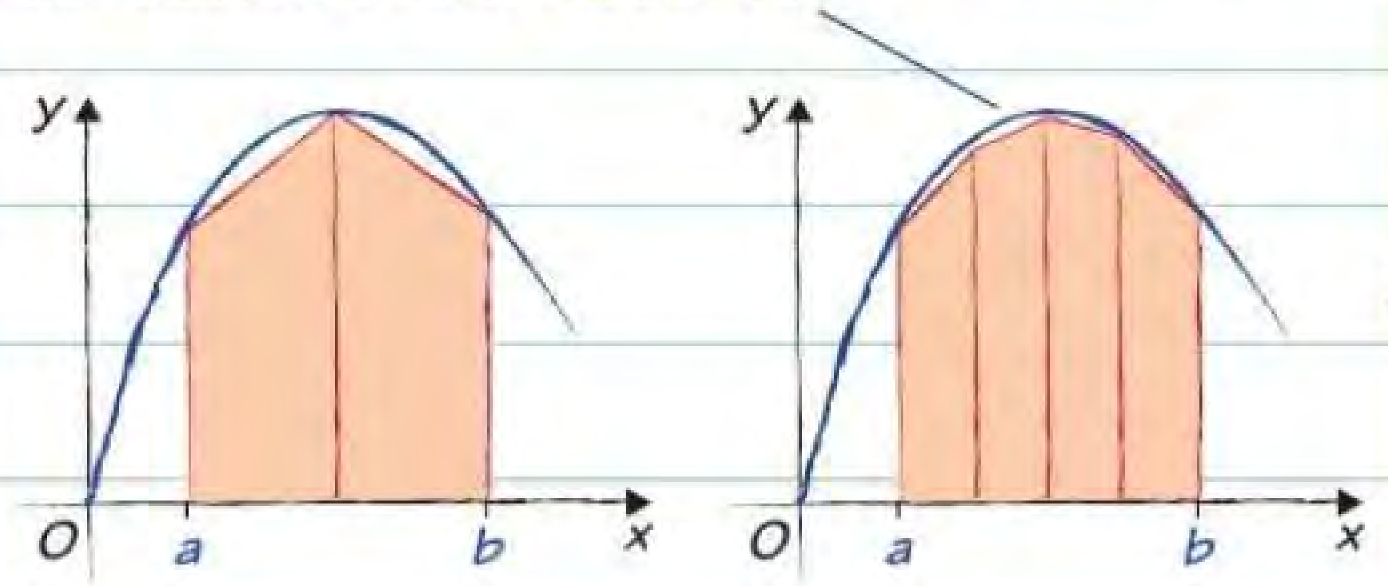
$$\frac{0.8859 - \ln(1 + \sqrt{2})}{\ln(1 + \sqrt{2})} \times 100\% = 0.51\% \text{ (2 d.p.)}$$

The value for $\frac{3\pi}{16}$ is given to 5 d.p. so make sure you give your answers to the same degree of accuracy.

Increasing accuracy

The trapezium rule becomes more accurate as you increase the number of strips used.

The tops of the trapezia are **closer** to the curve so the estimate is **more accurate**.



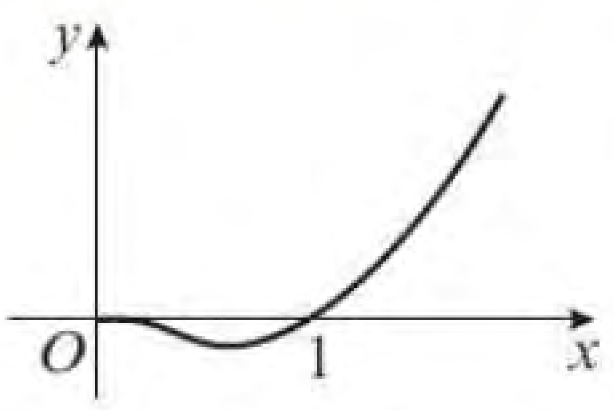
If you are asked to work out the **error** in an estimate use:

- Absolute error = Estimate − Exact value
- Percentage error = $\frac{\text{Estimate} - \text{Exact value}}{\text{Exact value}} \times 100\%$

Remember to change the width of each strip (h) for part (b)(ii).

Now try this

The diagram shows a sketch of the curve with equation $y = 3x^2 \ln x$, $x > 0$.



(a) Complete the table of values for values of y corresponding to $x = 1.25$, 1.5 and 1.75 Answer **(2 marks)**

x	1	1.25	1.5	1.75	2
y	0				8.31777

(b) Given that $I = \int_1^2 3x^2 \ln x\,dx$, use the trapezium rule

(i) with values of y at $x = 1$, $x = 1.5$ and $x = 2$ to find an estimate of I Answer

(ii) with all the values from the table to find another estimate of I. Answer **(5 marks)**

(c) Explain why increasing the number of values improves the accuracy of your estimate. **(1 mark)**

Answer